Math 221: Online Lecture Guidance

Satya Mandal

Spring 2020

1 Modeling Spread of the Virus

Date: 24 March 2020
The experts in governmental organizations, like Central Disease Control (CDC) or World Health Organization (WHO), have variety of models to make projections regarding spread of this virus. As an academic exercise, let us do out models.

Notations 1.1. Let t denote time. Let February 1 2020 (midnight) be $t = 0$. We may be thinking of World Population or US population.

1. Let P =total population size.

2. Let $y = y(t)$ denote the size of the infected (including those that are cured) population, at time t.

3. Also, let $Y_0 = y(0)$ be the size of infected population at $t = 0$.

1. **Exponential Growth Model** Most simple model would be to assume that rate of growth of the size $y(t)$ is proportional to the size itself. So, the model is

$$\frac{dy}{dt} = \lambda y \qquad \text{where } \lambda \text{ is assumed to be a constant.} \tag{1}$$

It follows,

$$y(t) = Y_0 e^{\lambda t}$$

2. The value of the parameter λ to be used, is determined from data on the rate of growth, in the recent past.

3. This model work fine for (very) short term projections.

4. For long term, exponential growth never works. That would mean extinction of the whole population.

5. The experience from the outbreaks in the past indicates, growth flattens. That would mean $\frac{dy}{dt}_{|t=t_0} = 0$ at some point of time $t = t_0$. Then, rate of growth declines.

6. So, it is reasonable conclude that the multiplier λ in the exponential growth (1) is not really a constant. I speculate $\lambda = \lambda(t)$ is a function of t, which decrease to **zero**. Better still, λ is a of something else, which is a function of t. In fact, as more and more people get infected, there are less and less to get infected.

7. I suggest that λ is a function of the proportion $\frac{P-y(t)}{P}$ of un-infected population. I suggest

   $$\lambda(t) = C\frac{P - y(t)}{P} \qquad \text{Note, as} y(t) \text{ increases } \lambda(t) decreases.$$

8. **Suggested Model:**

   $$\frac{dy}{dt} = C\frac{P - y}{P}y \tag{2}$$

   Here C is a constant, to be determined from data in the recent past.

Remark 1.2. I am not an expert in this business. Experts already have models that has worked in the past. They would readjust them, to deal with a new epidemic. For you and I, nothing stops us from trying our model (2), and fine tune it as data keeps coming.

2 Updating the model

About 20 hours since I wrote the model (2), I have been thinking about it (*without trying to find an answer from the internet*). The model is really simplistic. Following is my thoughts:

1. **First and foremost**, there are many unknown dependent variables:

$$\begin{cases} y_1(t) = & \text{size of the } \textbf{currently sick}, \textbf{ confirmed infected}, \text{ population} \\ y_2(t) = & \text{size of the currently infected population, } \textbf{without symtoms and unknown} \\ y_3(t) = & \text{size of the infected and } \textbf{recovered} \text{ population} \\ y_4(t) = & \text{size of the population who died from this} \\ y_5(t) = & \text{size of the } \textbf{uninfected} \text{ population} \end{cases}$$

   To start with, the model should look like:

$$\begin{cases} \frac{dy_1}{dt} = f_1(y_1, y_2, y_3, y_4, y_5, t) \\ \\ \frac{dy_2}{dt} = f_2(y_1, y_2, y_3, y_4, y_5, t) \\ \\ \frac{dy_3}{dt} = f_3(y_1, y_2, y_3, y_4, y_5, t) \\ \\ \frac{dy_4}{dt} = f_4(y_1, y_2, y_3, y_4, y_5, t) \\ \\ \frac{dy_5}{dt} = f_5(y_1, y_2, y_3, y_4, y_5, t) \end{cases} \tag{3}$$

Punch Line Today: The model (3) is the an example of **system of first order ODE**. We have to figure out model for f_1, f_2, f_3, f_4, f_5. Try to think, how would you model them?