§2.5 Rank of a Matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & & \cdots \\ a_{m1} & a_{m2} & a_{m3} & & a_{mn} \end{pmatrix}$$

① An r-Minor of A is ~~the~~ obtained/defined as follows:

ⓐ Delete $m-r$ rows and $n-r$ collumns from A

This way we get a square (sub) matrix M.

$|M| = \underline{\underline{\text{an}}}$ r-minor of A.

$$\text{rank}\, A = \max \left\{ r : \begin{array}{l} \text{there is an } r\text{-minor} \\ |M| \neq 0 \end{array} \right\}$$

Exercise: Suppose A is an $\boxed{n \times n}$-matrix,

(a) Assume A is nonsingular. Then $\boxed{\text{rank}\, A = n}$

(b) If A is singular, then $\text{rank}\, A < n$

(c) If $A \neq 0$ then $\text{rank}(A) \geq 1$.

Remark: Latter in the course, we will define rank of a Linear Transformation (of Vector spaces). That will be related to this.