Math 365: Lesson 3

Additional Problems from Probability

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1 Probability Problems of the First Type

Exercise 1.1 In a certain county, following is the distribution of population:

ethnicity	*W*	*H*	*AA*	*A*	*O*
percent	62	14	13	5	6

Here W=White, H=Hispanic, AA= African American, A= Asian, O=Others. A jury is selcted at random. So, the sample space is

$$S = \{W, H, AA, A, O\}$$

and the probabilty distribution is given by the above table.

1. What is the probability that the jury will a hispanic or asian? $Answer = P(H, A) = .14 + .05$.

2. What is the probability that the jury will not be an asian? $Answer = P(notW) = P(H, AA, A, O) = .14 + .13 + .05 + .06$.

Exercise 1.2 An arbitrary spot is selected in a swamp. The depth (in feet) of water in the swamp is has the following probability distribution:

depth	0+	1+	2+	3+	4+	5+	6+	7+	8+
probability	.1	.2	.09	.17	.13	.11	.08	.07	.05

So, here the sample space is

$$S = \{0+, 1+, 2+, 3+, 4+, 5+, 6+, 7+, 8+\}.$$

1. What is the probability that the depth at an arbitrary spot is less than three feet?

   $Answer = P(Less\ than\ 3) = P(0+, 1+, 2+) = .1 + .2 + .09 = .39$

2. What is the probability that the depth at an arbitrary spot is 3 feet or higher?

   $Answer = P(3\ feet\ or\ higher) = P(3+, 4+, \cdots, 8+) = .17 + .13 + .11 + .08 + .07 + .05 = .61$

Exercise 1.3 A Van pool can carry 7 people. Following is the distribution of number of riders in the van on a given day.

number of	1	2	3	4	5	6	7
probability	0	.12	.22	.23	.28	.08	.07

So, here the sample space is

$$S = \{1, 2, 3, 4, 5, 6, 7\}.$$

1. What is the probability that there will be at most 4 riders?

   $Answer = P(at\ most\ 4) = P(1, 2, 3, 4) = 0 + .12 + .22 + .23 = .57.$

2. What is the probability that there will be less than 4 riders?

   $Answer = P(less\ than\ 4) = P(1, 2, 3) = 0 + .12 + .22 = .34.$

3. What is the probability that there will be more than 4 riders?

   $Answer = P(more\ than\ 4) = P(5, 6, 7, 8) = .23 + .28 + .08 + .07 = .66.$

4. What is the probability that the vam will not be full on a particular day?

   $Answer = P(not\ full) = P(0, 1, 2, 3, 4, 5, 6) = 0 + .12 + .22 + .23 + .28 + .08 = .93$

Exercise 1.4 Following is the distribution of hourly wages (in whole dollars) earned by workers in an industry:

wage	7	8	9	10	11	12	13	14	15	16	17	18	19	20
probability	.04	.06	.07	.09	.11	.12	.14	.11	.09	.08	.04	.03	.01	.01

An employee is selected at random. So, here the sample space is

$$S = \{7, 8, 9, \ldots, 19, 20\}.$$

1. What is the probability that randomly selected worker makes less than 10 dollars an hour?

   $Answer = P(less\ than\ 10) = P(7, 8, 9) = .04 + .06 + .07 = .18$

2. What is the probability that randomly selected worker makes at least \$10 an hour?

   $Answer = P(at\ least\ 10) = P(10, 11, 12, \ldots, 19, 20) = .09 + .11 + .12 + \cdots + .01 + .01 = .82$

3. What is the probability that randomly selected worker makes between \$12-\$16 an hour?

   $Answer = P(12\ to\ 16) = P(12, 13, 14, 15, 16) = .12 + .14 + .11 + .09 + .08 = .54$

Exercise 1.5 In a school district, the distribution of number of students in a class has the following probability distribution:

number	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
prob	.03	.04	.06	.07	.10	.12	.13	.11	.09	.07	.06	.04	.03	.02	.02	.01

A child is selected at random from the school district. So, here the sample space is

$$S = \{8, 9, 10, \ldots, 19, 20, 21, 22, 23\}.$$

1. What is the probability that a child will be in a class of at least 20?

   $Answer = P(at\ least\ 20) = .03 + .02 + .02 + .01 = .08$

2. What is the probability that a child will be in a class of at most 10?

   $Answer = P(at\ most\ 10) = .03 + .04 + .06 = .13$

3. What is the probability that a child will be in a class of less than 10?

   $Answer = P(less\ than\ 10) = P(8, 9) = .03 + .04 = .07$

2 The Laws of Probability

Exercise 2.1 In a restaurant menu, entrees are served with rice product or potato product or others. Probability that an entree is served with rice product is .35 , probability that an entree is served with potato product is .40, probability that an entree is served with both is .15. What is the probability that an entree is served with either rice product or potato product?

Exercise 2.2 Usual infections can be bacterial or viral. Probability that a person will get a bacterial infection (next winter) is .35, probability that a person will get a viral infection is .65, probability that a person will get either a bacterial or a viral infection is .85.

1. What is the probability that a person will get both next winter?

2. What is the probability that a person will not get an infection next winter?

Exercise 2.3 You go for an examimation of upper stomach (EGD) and lower stomach (colonoscopy). Probability that some problem in upper stomach would be found is .15, probability that some problem in lower stomach would be found is .20 and probability that some problem both in lower and upper stomach would be found is .07.

1. What is the probobility that some problem would be found in either upper ot lower stomach?

2. What is the probobility that it would found that both upper and lower stomach are healthy?

Exercise 2.4 Probability that a person owns a domestic car is .55, probability that a person owns an import is .55 and the probability that a person owns both is .20.

1. What is the probability that a person wons either a domestic or an import?
2. Also what is the probability that a persone owns none?

(*Here essentially we are dealing with proportion of people who owns a domestic or import or so.*)

Exercise 2.5 In a county, 38 percent of the community is a minority. What is the probability that a randomly selected jury will not be a minority?

Solution: Let E be the event that the jury will be a minority. Then $P(E) = .38$ Therefore, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .38 = .62$$

Exercise 2.6 In a school district, probability that a student will be in a class of less than 10 students is .27. What is the probability that a randomly selected student will be a class 10 or more?

Solution: Let E be the event that the student will be in a class of less that 10 students. Then, $P(E) = .27$. So, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .27 = .73.$$

Exercise 2.7 In a swamp, probablity that the depth at a random spot is higher than 4 feet is .17. What is the probability that at a random spot, the depth is four feet or less?

Solution: Let E be the event that at a random spot the depth is higher than 4 feet. So, $P(E) = .17$. So, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .17 = .83.$$

Exercise 2.8 It is known that 43 percent of the work force in a town earns more than \$37,000 annualy. What is the probability that a randomly selected working person would make at most \$37,000 annualy?

Solution: Let E be the event that a randomly selected working person would make more than \$37,000 annualy. Therefore, $P(E) = .43$. Therefore, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .43 = .57.$$

Exercise 2.9 It is known that you can get an empty seat in the bus 64 percent of the rides. What is the probability that on a particular ride would not get a seat?

Solution: Let E be the event that you get an empty seat in the bus. So, $P(E) = .64$. Therefore, the answer is

$$P(not\ E) = 1 - P(E) = 1 - .64 = .36$$

3 The Rules of Counting

Ordered Selection

Exercise 3.1 Suppose there are 14 tennis players are competing is a tournament for the top three positions. How many outcomes are possible?

Solution: Since, order matters here, this is an ordered selection of 3 from 14. So, Answer

$$= {}_{14}P_3 = \frac{n!}{(n-r)!} = \frac{14!}{(14-3)!} = \frac{14!}{11!} = \frac{1.2.3.4.5 \cdots 13.14}{1.2.3 \cdots 11} = 12.13.14 = 2184.$$

Alternately, use TI-84 to compute ${}_{14}P_3$.

Exercise 3.2 Mathematics department has funds for awards for the best four teachers this year. Awards have cash values of \$5,000, \$3000, \$2000 and \$1000. There are 37 teachers in the math department. How many selection of four winners is possible?

Solution: Since, order matters here, this is an ordered selection of 4 from 37. So, Answer

$$={}_{37}P_4 = \frac{n!}{(n-r)!} = \frac{37!}{(37-4)!} = \frac{37!}{33!} = \frac{1.2.3\cdots36.37}{1.2.3\cdots33} = 34.35.36.37 = 1585080$$

Alternately, use TI-84 to compute ${}_{37}P_4$.

Unordered Selection

Exercise 3.3 Suppose there are 14 applications for three positions in the college office. The positions are all at the same level and pay. How many selection is possible?

Solution: Since all positions are alike, this is an unordered selection of 3 from 14. So,

$$Answer = {}_{14}C_3 = \frac{n!}{(n-r)!r!} = \frac{14!}{(14-3)!3!} = \frac{14!}{11!3!} = \frac{14.13.12}{1.2.3} = 364.$$

Alternately, use TI-84 to compute ${}_{14}C_3$.

Exercise 3.4 A soccer club has 17 players. Eleven has to be selected for a soccar match. How many selection is possible?

Solution: Since, order of selection is irrelevent, this is an unordered selection of 11 from 17.

$$={}_{17}C_{11}=\frac{n!}{(n-r)!r!}=\frac{17!}{(17-11)!11!}=\frac{17!}{6!11!}=\frac{12.13.14.15.16.17}{6!}=12376$$

Alternately, use TI-84 to compute ${}_{17}C_{11}$.

Exercise 3.5 A basket ball team has 21 players. Five are selected for a match. How many selection is possible?

Solution: Since, order of selection is irrelevent, this is an unordered selection of 21 from 5.

$$={}_{21}C_{5}=\frac{n!}{(n-r)!r!}=\frac{21!}{(21-5)!5!}=\frac{21!}{16!5!}=\frac{17.18.19.20.21}{5!}=20349.$$

Alternately, use TI-84 to compute ${}_{21}C_{5}$.

Exercise 3.6 Psychology department has funds for awards, of cash value \$2000 each, for best 5 teachers. There are 44 teachers in the math department. How many selection of five winners is possible?

Solution: Since, order of selection is irrelevent, this is an unordered selection of 5 from 44. So, Answer

$$={}_{44}C_{5}=\frac{n!}{(n-r)!r!}=\frac{44!}{(44-5)!5!}=\frac{37!}{39!5!}=\frac{40.41.42.43.44}{5!}=1086008$$

Alternately, use TI-84 to compute ${}_{44}C_{5}$.

Stand Alone use of Multiplication Rule

Exercise 3.7 Suppose there are three faculty positions in math department in statistics, algebra and geometry. For the statistics position there are 9 appilications, for the algebra position there are 12 appilications, for the geometry position there are 13 appilications. How many different ways is possible to fill these three positions?

Solution: The selection for these three positions have nothing to do with each other. So, permutaion (ordered selection) or combination (unordered selection) formulas do not apply. We use Multiplication Rule as a stand alone tool and make a table:

$position$	$number\ of\ choice$
$statistics$	9
$algebra$	12
$geometry$	13
$answer = product =$	1404

Exercise 3.8 A candidate for a elected position dresses carefully. In his wardrobe, he has 5 pants, 4 shirts, 7 jackets, 8 ties and 4 pairs of shoes. How may different ways he can dress?

Solution: He has to pick one of each item. These items are selected from seperate 'pots'. So, permutaion (ordered selection) or combination (unordered selection) formulas do not apply. We use Multiplication Rule as a stand alone tool and make a table:

$Item$	$number\ of\ choice$
$pants$	5
$shirts$	4
$jackets$	7
$ties$	8
$answer = product =$	1120

Exercise 3.9 A football team has 43 offense player and 38 defense players. Eleven from each needs to be selected for a game. How many selection is possible for such a game.

Solution: Offense and defense playesr are selected from seperate 'pots'. We use Multiplication Rule as a stand alone tool and make a table:

$select\ 11\ offense\ from\ 43$	${}_{43}C_{11} = 5752004349$
$select\ 11\ defense\ from\ 38$	${}_{38}C_{11} = 1203322288$
$answer = product =$	$too\ big$

4 Independent Events

Exercise 4.1 Suppose you went for a job interview in Lawrence and another one in Kansas City. Probability of that you will get the job in Lawrence is .25 and the probability of that you will get the job in Kansas City is .33. It is reasonable to assume independence.

1. What is the probability that you will get both the jobs?

   Solution: Let E be the event that get the job in Lawrence and F be the event that get the job in Kansas City. So,

   $$P(E) = .25 \qquad and \qquad P(F) = .33$$

   Answer: $P(Both) = P(E \text{ } and \text{ } F) = P(E)P(F) = .25 * .33 = .0825$

2. What is the probability that you will get neither?

   Solution: We have

   $P(not \text{ } E) = 1 - P(E) = 1 - .25 = 75, \qquad and \qquad P(not \text{ } F) = 1 - .33 = .67$

   Answer: $P(Neither) = P((not \text{ } E) \text{ } and \text{ } (not \text{ } F)) = P(not \text{ } E)P(not \text{ } F) = .75 * .67 = .5025$

Exercise 4.2 You are taking the Topic course in KU and your brother is taking the same course in MU. The probability that you will get an A is .18 and the probability that your brother will get an A is .21.

1. What is the probability that both of you will get and A.

   Solution: Let E be the event that you will get an A and F be the event that your brother will get an A. Then,

   $$P(E) = .18 \qquad and \qquad P(F) = .21.$$

   Answer: $P(Both) = P(EandF) = P(E) * P(F) = .18 * .21 = .0378.$

2. What is the probability that none of you will get an A. We have

$$P(not\ E) = 1 - .18 = .82 \qquad and \qquad P(not\ F) = 1 - .21 = .79.$$

Answer: $P(Neither) = P((not\ E)and(not\ F)) = P(not\ E){*}P(not\ F)) = .82 * 79 = .6478$

Exercise 4.3 Probability that you will receive a call from a sibling this week is .35 and the that you will receive a call from a parent this week is .43. What is the probability that you receive a call from both, this week. (Assume independence.)

Solution: Let E be the event that you will receive a call a sibling this week and F be the event that you will receive a call a parent. Then

$$P(E) = .35 \qquad and \qquad P(F) = .43.$$

Answer: $P(Both) = P(E\ and\ F) = P(E) * P(F) = .35 * .43 = .1505.$

Exercise 4.4 Probability that it will rain in Lawrence today is .22 and probability that it will rain today at your home town is .40. What is the probability that it will rain in both places?

Solution: Let E be the event that it will rain in Lawrence today and F be the event that it will rain at your home town today.

Then

$$P(E) = .22 \qquad and \qquad P(F) = .40.$$

Answer: $P(Both) = P(E\ and\ F) = P(E) * P(F) = .22 * .40$

Exercise 4.5 According to the poll, probability that a person would vote for Candidate-A is .43.

1. What is the probability that both you and I would vote for Candidate-A? (We can assume independence because you and I do not influence each other.)

   Solution: Let E be the event that you will vote for Candidate-A and let F be the event that I will vote for Candidate-A. Then $P(E) = P(F) = .43$.

   Answer: $P(Both) = P(E\ and\ F) = P(E) * P(F) = .43 * .43 = .1849$

2. What is the probability that neither you nor I would vote for Candidate-A?

   Solution: We have

   $$P(not\ E) = 1 - .43 = .57 \qquad and \qquad P(not\ F) = 1 - .43 = .57$$

   Answer: $P(Neither) = P((not\ E)\ and\ (not\ F)) = P(not\ E) * P(not\ F) = .57 * .57 = .3249$

Exercise 4.6 Following is some statistics about pneumonia vaccine.

1. Probability that an individual will not get a pneumonia vaccine shot is .88.

2. Probability that an individual will not get a Pneumonia shot and will get pneumonia in winter is .04.

3. Probability that an individual will get pneumonia in winter is .03.

What is the conditional probability that a randomly selected person will get pneumonia given that he/she did not take pneumonia shot?

Exercise 4.7 Probability that there will be shortage of flu vaccine is 0.05. Probability that unemployment rate will drop below four percent is 0.12. What is the probability that there will be a shortage of flu vaccine and unemployment rate will fall below four percent? (Obviously, these are independent events.)