Math 365: Elementary Statistics
Homework and Problems (Solutions)

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Contents

Chapter 1

The Language and Terminology

1.1   The Language and Terminology

1. Consider the following is data on the 9-month salary of mathematics faculty members (to the nearest thousand dollars) in the year 1999-00:

$$
\begin{array}{cccccccccc}
61 & 63 & 72 & 80 & 70 & 76 & 97 & 60 & 57 & 65 \\
67 & 73 & 66 & 75 & 61 & 54 & 61 & 65 & 70 & 68 \\
53 & 50 & 50 & 40 & 51 & 44 & 62 & 57 & 51 & 52 \\
50 & 57 & 52 & 52 & 56 & 46 & & & &
\end{array}
\tag{1.1}
$$

Use class width = 10K. Complete the following frequency table:

$Class$	$39.5 - 49.5$	$49.5 - 59.5$	$59.5 - 69.5$	$69.5 - 79.5$	$79.5 - 89.5$	$89.5 - 99.5$
$Freq$						

2. Following is the grand total (out of 400) of scores obtained by students in a class.

$$
\begin{array}{cccccccccc}
386 & 343 & 287 & 394 & 303 & 280 & 333 & 389 & 376 & 350 \\
388 & 380 & 320 & 391 & 371 & 354 & 366 & 354 & 284 & 298 \\
327 & 386 & 380 & 370 & 363 & 382 & 362 & 384 & 343 & 352 \\
350 & 391 & 345 & 385 & 310 & 380 & 381 & 362 & 326 & 82 \\
391 & 328 & 345 & 376 & & & & & &
\end{array}
\tag{1.2}
$$

We use class width $= 50$. Complete the frequency table of the date data in (1.2):

Class	Frequency
$51 - 100$	
$101 - 150$	
$151 - 200$	
$201 - 250$	
$251 - 300$	
$301 - 350$	
$351 - 400$	

If a data value falls on the boundary, count it on the left interval.

3. The following is data on the weight (in ounces), at birth, of some babies.

$$
\begin{array}{cccccccccc}
74 & 105 & 124 & 110 & 119 & 137 & 96 & 110 & 120 & 115 & 140 \\
65 & 135 & 123 & 129 & 72 & 121 & 117 & 96 & 107 & 80 & 91 \\
74 & 123 & 124 & 124 & 134 & 78 & 138 & 106 & 130 & 97 & 145 \\
93 & 133 & 128 & 96 & 126 & 124 & 125 & 127 & 62 & 127 & 92 \\
95 & 118 & 126 & 94 & 127 & 121 & 117 & 124 & 93 & 135 & 156 \\
143 & 125 & 120 & 147 & 138 & 72 & 119 & 89 & 81 & 113 & 91 \\
133 & 127 & 138 & 122 & 110 & 113 & 100 & 115 & 110 & 135 & 141 \\
97 & 127 & 120 & 110 & 107 & 111 & 126 & 132 & 120 & 108 & 148 \\
143 & 103 & 92 & 124 & 150 & 86 & 121 & 98 & 74 & 85 & 99 \\
\end{array}
$$

(1.3)

Complete the frequency table of the weight distribution data in (1.3):

Class	Frequency
$60.5 - 70.5$	
$[70.5 - 80.5$	
$80.5 - 90.5$	
$90.5 - 100.5$	
$100.5 - 110.5$	
$110.5 - 120.5$	
$120.5 - 130.5$	
$130.5 - 140.5$	
$140.5 - 150.5$	
$150.5 - 160.5$	

Chapter 2

Measures of Central Tendency and Measures of Dispersion

2.1   Mean and Median

1. The following is the price (in dollars) of a stock (say, CISCO SYS-TEMS) checked by a trader several times on a particular day.

    $$139 \quad 143 \quad 128 \quad 138 \quad 149 \quad 131 \quad 143 \quad 133$$

    (a) Find the mean price (in dollars) observed by the trader.
    (b) Find the median price observed by the trader (in dollars).

2. The following figures refer to the GPA of six students:

    $$3.0 \quad 3.3 \quad 3.1 \quad 3.0 \quad 3.1 \quad 3.1$$

    (a) Find the median of the GPA.
    (b) Find the mean of the GPA.

3. The following data give the lifetime (in days) of certain light bulbs.

    $$138 \quad 952 \quad 980 \quad 967 \quad 992 \quad 197 \quad 215 \quad 157$$

    (a) Find the mean for the lifetime of these light bulbs.

    (b) Find the median for the lifetime of the bulbs.

4. An athlete ran an event 32 times. The following frequency table gives the time taken (in seconds) by the athlete to complete the events.

Time (*in seconds*)	Frequency
11.6	4
11.7	5
11.8	6
11.9	7
12.0	6
12.1	4
Total	32

    (a) Compute the mean time taken by the athlete. (Write up to 4 significant digits.)

    (b) Find the median time taken by the athlete

5. The following are the weights (in ounces), at birth, of 30 babies born in Lawrence Memorial Hospital in May 2000.

$$
\begin{array}{cccccccccc}
94 & 105 & 124 & 110 & 119 & 137 & 96 & 110 & 120 & 115 \\
104 & 135 & 123 & 129 & 72 & 121 & 117 & 96 & 107 & 80 \\
96 & 123 & 124 & 124 & 134 & 78 & 138 & 106 & 130 & 97
\end{array}
$$

    (a) Compute the mean weight, at birth, of the babies.

    (b) Using the previous table, compute the median weight, at birth, of the babies.

6. Following is a frequency table for the hourly wages (paid only in whole dollars) of 99 employees in an industry.

Hourly Wages	7	8	9	10	11	12	13	14	15	16	17	18	19
Frequency		1	4	10	4	9	8	6	5	17	19	3	1	2

    (a) Compute the mean hourly wage.

    (b) Using the previous frequency table, compute the median hourly wage.

7. The following is the frequency table on the number of typos found in a sample of 30 books published by a publisher.

Number of Typos	156	158	159	160	162
Frequency	6	4	5	6	9

   (a) Compute the mean number of typos in a book.

   (b) Using the previous frequency table, compute the median number of typos found in a book.

8. The following are the lengths (in inches), at birth, of 14 babies born in Lawrence Memorial Hospital in May 2000.

Length	17	18.5	19	20	21.5
Frequency	2	3	4	3	2

   (a) Compute the mean length, at birth, of these babies.

   (b) Using the previous table, compute the median length, at birth, of these babies.

2.2 Variance and Standard Deviations

1. The following is the price (in dollars) of a stock (say, CISCO SYSTEMS) checked by a trader several times on a particular day.

$$138 \quad 142 \quad 127 \quad 137 \quad 148 \quad 130 \quad 142 \quad 133$$

   (a) Find the variance of the price (to four decimal places).

   (b) Find the standard deviation of the price (to four decimal places).

2. The following figures refer to the GPA of six students:

$$3.0 \quad 3.3 \quad 3.1 \quad 3.0 \quad 3.1 \quad 3.1$$

   (a) Find the variance of the GPA (to four decimal places).

   (b) Find the standard deviation of the GPA (to four decimal places).

3. The following data give the life time (in days) of certain light bulbs.

938 952 980 967 992 997 915 957

(a) Find the variance for the life time of these bulbs (to four decimal places).

(b) Find the standard deviation for these bulbs (to four decimal places).

4. An athlete ran an event 32 times. The following frequency table gives the time taken (in seconds) by the athlete to complete the events.

Time (*in seconds*)	Frequency
11.6	4
11.7	5
11.8	6
11.9	7
12.0	6
12.1	4
Total	32

(a) Compute the variance for the times taken by the athlete (to four decimal places).

(b) Find the standard deviation for the times taken by the athlete (to four decimal places).

5. The following are the weights (in ounces), at birth, of 30 babies born in Lawrence Memorial Hospital in May 2000.

94   105 124 110 119 137 96   110 120 115
104 135 123 129 72   121 117 96   107 80
96   123 124 124 134 78   138 106 130 97

(a) Compute the variance of the weight, at birth, of the babies.

(b) Compute the standard deviation of the weight, at birth, of the babies.

6. Following is a frequency table for the hourly wages (paid only in whole dollars) of 99 employees in an industry.

Hourly Wages	7	8	9	10	11	12	13	14	15	16	17	18	19
Frequency	11	4	10	4	9	8	6	5	17	19	3	1	2

(a) Compute the variance of the hourly wage.

(b) Compute the standard deviation of the hourly wage.

7. The following is a frequency table on the number of typos found in a sample of 30 books published by a publisher.

Number of Typos	156	158	159	160	162
Frequency	6	4	5	6	9

(a) Compute the variance of the number of typos in a book.

(b) Compute the standard deviation of the number of typos in a book.

8. The following are the lengths (in inches), at birth, of 14 babies born in Lawrence Memorial Hospital in May 2000.

Length	17	18.5	19	20	21.5
Frequency	2	3	4	3	2

(a) Compute the variance of the length, at birth, of these babies.

(b) Compute the standard deviation of the length, at birth, of these babies.

Chapter 3

Probability

3.1 Basic Concept of Probability

None

3.2 Probability Table and Equally likely

1. The following table gives the probability distribution of a loaded die.

Face	1	2	3	4	5	6
Probability	0.20	0.15	0.15	0.10	0.05	0.35

    Find the probability that an even number face will show up when you roll the die.

    Solution:
    $$P(\{2, 4, 6\}) = .15 + .10 + .35 = .60$$

2. A die is rolled twice. What is the probability that the two numbers are unequal?

    Solution: $n(S) = 36$
    Let $E =$ that the two numbers are unequal

So, $(not\ E) = (1,1,),(2,2),(3,3),(4,4),(5,5),(6,6)$
So, $n(not\ E) = 6$
So, $n(E) = n(S) - n(notE) = 36 - 6 = 30$
$P(E) = \frac{n(E)}{n(S)} = \frac{30}{36} = .8333$

3. A student wants to pick a school based on the recent grade distribution
    of the school. Following is the grade distribution of the previous year
    in a school:

Grades	A	B	C	D	F
Percentage of Students	19	33	31	14	3

    What is the probability that a student will receive a C-grade or higher
    in this school?

    Solution: Let $E = \{C, B, A\}$ =event that the selected student will
    have a C-grade or higher
    $P(E) = .19 + .33 + .31 = .83$

4. A container has 11 red balls, 15 green balls, 10 white balls, and 8 yellow
    balls. A child picks up a ball from the container. Find the probability
    that the ball is red.

    Solution: Let $E = \{Red\}$ =event that boy will pick up a red ball.
    $P(E) = \frac{n(E)}{n(S)} = \frac{11}{33} = .3235$

5. A die is rolled twice. What is the probability that the product of the
    two numbers is 11?

    Solution: *Answer* $= 0$

6. A magician asks you to pick a card from a deck of 52 cards. What is
    the probability that you will pick a heart?

    Solution: *Answer* $= \frac{13}{52} = .25$

7. You toss a coin 4 times. What is the total number of outcomes?

    Solution: *Answer* $= 2^4 = 16$

8. The sex of 4 unborn children is to be determined in a lab today. What is the probability that there will be at least 2 girls?

   Solution: $n(S) = 2^4 = 16$
   Let $E =$ be the even that there would be at least two girls
   $(not\ E) =$ means 0 or 1 girl
   So, $(not\ E) = \{BBBB, GBBB, BGBB, BBGB, BBBG\}$
   $n(not\ E) = 5$
   $n(E) = n(S) - n(not\ \text{E})$=16-5=11
   So, $P(E) = \frac{n(E)}{n(S)} = 11/16$

3.3   Laws of Probability

1. Suppose E and F are two events. It is given that P(E)=.22, P(not F)=.65, and P(E and F)=.13. Determine the probability that either E or F occur.

   Solution: First, $P(F) = 1 - P(not\ F) = 1 - .65 = .35$
   $P(E\ or\ F) = P(E) + P(F) - P(E\ and\ F) = .22 + .35 - .13 = .44$

2. Suppose E and F are two events. It is given that P(not E)=.33, P(not F)=.54, and P(E or F)=.68. Determine the probability that both E and F occur.

3. Use the information in Question 2. Determine the probability that neither E nor F occur.

4. The proportion of students who own a vehicle is 0.55, and the proportion of students who live in a dorm is 0.41. If the proportion of students who either own a vehicle or live in a dorm is 0.83, find the proportion of students who live in a dorm and own a vehicle.

5. The probability that it does not rain in Arizona in June is 0.22. Find the probability that it rains at least once in the month of June in Arizona.

Solution: Let E = event that it rains at least once in June.
(*not E*) = event that it does not does not rain in June
$P(not\ E) = .22$
$P(E) = 1 - P(not\ E) = 1 - .22 = .78$

6. The probabilities that a particular office phone rings 0, 1, 2, 3 times in half hour are respectively: .05, .15, .21, .42. Find the probability that the phone rings at least 4 times between 3:30-4:00 pm.

   Solution: E =event that it rings at least 4 times.
   So, (*not E*) $= \{0.1, 2, 3\}$.
   $P(not\ E) = P(0, 1, , 2, 3) = P(0) + P(1) + P(2) + P(3) = .05 + .15 + .21 + .42 = .83$.
   So, $P(E) = 1 - P(not\ E) = 1 - .83 = .17$.

7. The probability of a student going to a bar on Wednesday is 3/5. The probability of the student going to class on Thursday is 4/5. If the probability that the student either goes to the bar or goes to class the next day is 9/10, what is the probability that the student will do the both?

8. The probability that a student will major in mathematics is .13 and the probability that a student will have a major in engineering is .31. The probability that a student will double major in mathematics and engineering is .07. What is the probability that a student will major in mathematics or engineering?

3.4   Counting Techniques and Probability

1. Compute $_8C_2$.

2. Suppose 2 scholarships, $1000 each, is to be given to 2 students. There are 9 applicants. How many choices of 2 awardees are possible?

   Solution: Each scholarship is worth same amount.
   So, order of selection does not count.
   It is an unordered selection.

So, answer=$_9C_2 = n!/[(n-r)!r!] = 9!/[7!2!] = 9*8/[1*2] = 36$.

3. Suppose 2 scholarships, one for \$2000 and another for \$1000, will be given to 2 students. There are 9 applicants. How many choices of 2 awardees are possible?

   Solution: Scholarships have distinct values.
   So, order of selection has to be taken into account.
   So, Answer=$_9P_2 = n!/(n-r)! = 9!/7! = 8*9 = 72$.

4. In an annual sports meet, 9 students compete in 2 events (the 100 meter and 1000 meter race). How many ways can two winners can be picked?

   Solution: Repetitions are allowed here.
   Use multiplication principle:
   First event : 9 choice
   2nd event : 9 choice
   Answer= product = $9*9 = 81$

5. A coin is tossed 6 times. What is the total number of possible outcomes?

   Solution: answer=$2*2*2*2*2*2 = 64$

6. The sex of 8 unborn children is to be determined today in a lab. What is the probability that 2 particular children (say the child of Elizabeth and the child of Maria) will be boys?

   Solution: Each child could be B or G.
   $n(S) = 2*2*2*2*2*2*2*2 = 2^8$
   $n(E) = 1*1*2*2*2*2*2*2 = 2^6$
   $P(E) = \frac{n(E)}{n(S)} = \frac{2^6}{2^8} = .25$

7. A committee of 8 is to be formed from a group of 7 mothers, 6 fathers, and 8 children. What is the probability that the committee will have exactly 4 fathers?

Solution: The sample space consists of all possible committees of 8, from total of $n = 7 + 6 + 8 = 21$ people. Committee selection is an unordered selection.

The number of such committees $n(S) =_{21} C_8$.

Let $E =$ be the event that there would be 4 fathers

(and hence 4 others from $7 + 8 = 15$).

So, $n(E) = (_6 C_4)(_{15} C_4)$

So, $P(E) = \frac{n(E)}{n(S)} = \frac{(_6 C_4)(_{15} C_4)}{_{21} C_8}$.

8. The 6 seats in the first row are to be assigned to a class of 17 women and 20 men. What is the probability that all the seats will be assigned to women.

   Solution: Seat assignment is an ordered selection.

   Total number of such assignment is

   $n(S) =_{37} P_6$.

   Let $E =$ be the event that all seat would be assigned to women.

   $n(E) =_{17} P_6$

   So, $P(E) = \frac{n(E)}{n(S)} = \frac{_{17} P_6}{_{37} P_6}$.

3.5   Conditional Probability and Independence

1. For two events, E and F, it is given that the probabilities $P(E) = 0.6$, $P(F|E) = 0.3$, and $P(F) = 0.7$. What is the probability that E and F occur simultaneously?

   Solution: $P(E \ and \ F) = P(F|E)P(E) = (0.3) * (0.6)$.

2. For two events, E and F, it is given that the probabilities $P(E) = 0.5$, $P(F \ and \ E) = 0.3$, and $P(F) = 0.9$. What is the conditional probability that F occurred, given that E has occurred?

   Solution: $P(F|E) = P(F \ and \ E)/P(E) = 0.3/.05$

3. Following are some data from a hospital emergency room:

   (a) The probability that a patient in the emergency room will have health insurance is 0.8.

(b) The probability that a patient in the emergency room will survive the treatment 0.9.

(c) The probability that a patient in the emergency room will have health insurance and will also survive is 0.6.

What is the conditional probability that a patient in the emergency room will survive, given that he/she has health insurance.

Solution: Let H = the event that the patient has Health insurance.
Let E = the event that the patient will survive.
Given $P(H) = 0.8.$, $P(E) = 0.9$, $P(H \ and \ E) = 0.6$
$P(E|H) = \frac{H \ and \ E}{P(H)} = \frac{0.6}{0.8}$

4. Following are some observations on the stock market:

   (a) The probability that the Dow Jones Industrial Average (DJIA) will go up today is 0.75.

   (b) The probability that the value of your portfolio will go up today is 0.80.

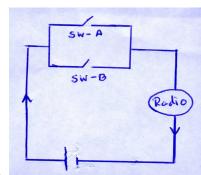   (c) The probability that both the DJIA will go up and the value of your portfolio will go up today is 0.6.

   What is the conditional probability that the value of your portfolio went up today, given that the DJIA went up today?

   Solution: Let D = the event that DJIA will go up.
   Let E = the event that your portfolio will go up.
   Given $P(D) = 0.75.$, $P(E) = 0.80$, $P(D \ and \ E) = 0.6$
   $P(E|D) = \frac{D \ and \ E}{P(D)} = \frac{0.6}{0.75.}$

5. Following are some data on allergy treatment:

   (a) The probability that a person will get the allergy in a season is 0.55.

   (b) The probability that a person has taken an allergy treatment and gets an allergy in the season is 0.15.

   (c) The probability that a person has taken the treatment is 0.6.

What is the conditional probability that a person will get an allergy, given that she/he has taken the allergy treatment?

Solution: Let E = the event one would get the allergy in a season.
Let F = the event that one would have taken an allergy treatment.
Given $P(E) = 0.55.$, $P(F) = 0.6$, $P(F \text{ and } E) = 0.15$
$P(E|F) = \frac{E \text{ and } F}{P(F)} = \frac{0.15}{0.6}$

6. Consider the following circuit with the radio: It is given that the probability that switch A is closed is 0.35 and the probability that switch B is closed is 0.40. It is also known that the 2 switches function independently. What is the probability that the radio is playing?

Solution: E =event that switch-A is on.
F =event that switch-B is on. We compute $P(E \text{ or } F)$.
We have $P(E) = 0.35$, $P(F) = 0.40$.
$P(E \text{ and } F) = P(E)P(F) = .35 * 40 = .14.$
(Because of Independence.)
$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F) = .35 + .40 - .14 = .61.$

7. An airplane has 2 engines and the engines function independently. The probability that the first engine fails during a flight is 0.1 and the probability that the second engine fails is 0.1. What is the probability that both will fail during a flight?

Solution: E =event that Engine-1 is fails.
F =event thatEngine-1 is fails.
We have $P(E) = 0.1$, $P(F) = 0.1$.
We compute $P(E \text{ and } F)P(E)P(F) = (.1) * (.1)$

8. The probability that you will receive a wrong number call this week is 0.2. The probability that you will receive a sales call this week is 0.5. The probability that you will receive a survey call this week is 0.3. What is the probability that you will receive one of each this week? (Assume independence.)

   Solution: Let $E =$ be the event that you receive a wrong number call
   Let $F =$ be the event that you receive a sales call
   Let $G =$ be the event that you receive a survey call
   $P(E \ and \ F \ and \ G) = P(E)P(F)P(G) = (0.2.) * (0.5) * (0.3)$

9. The probability that you will receive a wrong number call this week is 0.1. The probability that you will receive a sales call this week is 0.6. What is the probability that you will receive either a wrong number call or a sales call this week? (Assume independence.)

   Solution: Let $E =$ be the event that you receive a wrong number call
   Let $F =$ be the event that you receive a sales call
   $P(E) = 0.1, \ P(F) = 0.6.$
   $P(E \ or \ F) = P(E) + P(F) - P(E \ and \ F) = P(E) + P(F) - P(E)P(F) = 0.1 + 0.6 + (0.1) * 0.6).$

10. During your trip to Florida, the probability that the weather in Florida will be good is 0.9 and the probability that it will be cold in Lawrence is 0.3. What is the probability that, during your trip, you will have good weather in Florida and it will be cold in Lawrence? (Assume independence.)

    Solution: Let $E =$ be the event that weather in Florida will be good
    Let $F =$ be the event that it will be cold in Lawrence
    $P(E \ and \ F) = (0.9) * (0.3).$

Chapter 4

Random Variables

4.1 Random Variables

None

4.2 Probability Distribution

1. The following table gives the probability distribution of the starting salary X earned by new graduates from some university (to the nearest $10K).

$X = x$	0	1	2	3	4	5	6	7
$p(x)$	0.05	0.15	0.22	0.22	0.17	0.10	.05	.04

   What is the expected starting salary of a new graduate?

   Solution: $E(X) = \mu = \sum x_i p(x_i) =$
   $= 0*.05 + 1*.15 + 2*.22 + 3*.22 + 4*.17 + 5*.10 + 6*.05 + 7*.04 = 3.01$

2. For the starting salary X in Question 1, what is the standard deviation of the staring salary?

Solution: Variance
$\sigma^2 == \sum x_i^2 p(x_i) - \mu^2 =$
$[0 * .05 + 1 * .15 + 4 * .22 + 9 * .22 + 16 * .17 + 25 * .10 + 36 * .05 + 49 * .04] - (3.01)^2 = 2.9299$
So, standard Deviation $\sigma = \sqrt{\sigma^2} = \sqrt{2.9299} = 1.7117$

3. Maria is a plumber who works for 3 different employers. Employer A pays her \$120 a day, employer B pays her \$70 dollars a day, and employer C pays her \$180 a day. She works for whoever calls her first. The probability that employer A calls her first is 0.4, the probability that employer B calls first is .2, and the probability that employer C calls her first is 0.3 (the probability that no one calls is .1). What is the expected income of Maria per day?

   Solution: Let $X =$ Income of Maria on a day.
   So, X takes values 0, 120, 70, 180. The probability function is given by $p(0) = .1$, $p(120) = .4$, $p(70) = .2$, $p(180) = .3$.
   In a tabular form, the distibution of Maria's income is given by

$X = x$	0	120	70	180
$p(x)$	.1	.4	.2	.3

   So, mean $E(X) = \mu = \sum x_i p(x_i) =$
   $0 * p(0) + 120 * p(120) + 70 * p(70) + 180 * p(180)$
   $= 0 * 0 + 120 * .4 + 70 * .2 + 180 * .3 = 116$

4. Maria is the plumber in Question 3. What is the standard deviation of the daily income of Maria?

   Solution: Variance $\sigma^2 = s \sum x_i^2 p(x_i) - \mu^2$
   $= [0 * 0 + 14400 * .4 + 4900 * .2 + 32400 * .3] - (116)^2 = 3004$
   So, standard Deviation $\sigma = \sqrt{\sigma^2} = \sqrt{3004}$

5. The number X of typos in a website has the following probability distribution.

$X = x$	0	1	2	3	4	5
$p(x)$	.14	0.27	0.27	0.18	0.09	0.05

   What is the expected number of typos in a website?

Solution: The mean
$\mu = \sum x_i p(x_i) = 0*p(0)+1*p(1)+2*p(2)+3*p(3)+4*p(4)+5*p(5)$
$== 0*.14 + 1*.27 + 2*.27 + 3*.18 + 4*.09 + 5*.05 = 1.96$

6. Let X be the number of typos, as in Question 5. What is the standard deviation of X?

   Solution: Variance $\sigma^2 = \sum x_i^2 p(x_i) - \mu^2$
   $= (0*p(0)+1*p(1)+4*p(2)+9*p(3)+16*p(4)+25*p(5)]-(1.96)^2$
   $= (0*.14+1*.27+4*.27+9*.18+16*.09+25*.05)-(1.96)^2 =$
   $5.66 - 4.8416 = 1.8184$

   So, standard Deviation $\sigma = \sqrt{\sigma^2} = \sqrt{1.8184}$

7. Let X be the number of typos, as in Question 5. What is the probability that there would be at least 3 typos is a book?

   Solution:
   $P(3 \le X) = P(X = 3) + P(X = 4) + P(X = 5) = 0.18 + 0.09 + 0.05$

8. Let X represents the number of students in a class, in a school district. The following is the distribution of X:

$X = x$	11	12	13	14	15	16	17	18	19	20	21
$p(x)$	.08	.09	.11	.13	.14	.13	.10	.09	.06	.05	.02

   What is the expected number of students in a class?

   Solution:

   $$E(X) = \mu = \sum x_i p(x_i) = 11*.08 + 12*.09 + 13*.11 + \cdots = 15.27$$

9. Refer to Question 8. Compute the standard deviation σ.

   Solution: The varience

   $$\sigma^2 = \left(\sum x_i^2 p(x_i)\right) - \mu^2 = \left((11^2*.08 + 12^2*.09 + 13^2*.11 + \cdots\right) - (15.27)^2$$
   $$= 240.03 - 233.1729 = 6.8571$$

   So, the St. deviation

   $$\sigma = \sqrt{\sigma^2} = \sqrt{6.8571} = 2.6186$$

10. A Gambling house has a loaded die. One pays \$3 to play each time. One gets back number of dollars equal to the face that turns up. The following is the probability distribution of the loaded die, together with the win X.

$Face$	1	2	3	4	5	6
$p(x)$		.3	.2	.2	.1	.1	.1
$Win\ X = x$	-2	-1	0	1	2	3

Compute the expected win $E(X)$?

Solution: The mean,
$E(X) = \mu = \sum x_i p(x_i) = (-2)*.3+(-1)*.2+0*.2+1*.1+2*.1+3*.1 = -2$

4.3   The Bernoulli and Binomial Experiments

1. The statistics of campus interviews show that for every 10 interviews 4 job offers are made. Out of 66 interviews, what is the probability that at least 30 offers would be made? (Use TI-84, Silver Edition.)

Solution: Here $n = 66$, $p = \frac{4}{10} = .4$
$P(30 \le X) = 1 - P(X \le 29)$
$= 1 - binomialcdf(66, .4, 29) = 1 - .7829 = .2171$

2. Refer to Question 1. What would be the expected number of job offers made?

Solution: mean $\mu = E(X) = np = 66 * .4 = 26.4$

3. Refer to Question 1. What would be the standard deviation σ of the number of job offers made?

Solution: Standard Deviation
$\sigma = \sqrt{np(1-p)} = \sqrt{66 * .4 * (1 - .6)} = 3.9799$

4. Assume that 20 percent of the babies get fever after immunization shoots. If 130 babies are immunized in a clinic, what is the probability that at most 30 will get fever?

Solution: Here $n = 130$, $p = .2$
$P(X \leq 30) = binomialcdf(130, .2, 30) = .8384$

5. Refer to Question 4. What is the expected number of babies that would get fever?

Solution: mean $\mu = E(X) = np = 130 * .2 = 26$

6. Refer to Question 4. What would be the standard deviation σ ?

Solution: Standard Deviation
$\sigma = \sqrt{np(1-p)} = \sqrt{130 * .2 * (1 - .2)} = 4.5607$

7. It is assumed by the coach that a swimmer can finish an event within target time 45 percent times during practice. Suppose the swimmer swims 80 times. What is the probability that he/she will finish within the target time at least 40 times?

Solution: Here $n = 80$, $p = .45$
$P(40 \leq X) = 1 - P(X \leq 39) = 1 - binomialcdf(80, .45, 39)$
$= 1 - .7846 = .2154$

8. Refer to Question 8. What is the expected number $E(X)$ of times that he/she will finish within target time?

Solution: mean $\mu = E(X) = np = 80 * .45$

9. Refer to Question 8. What is the standard deviation σ of the number of times X that he/she will finish within target time?

Solution: Standard Deviation
$\sigma = \sqrt{np(1-p)} = \sqrt{80 * .45 * (1 - .45)}$

10. It was reported, that 20 percent of the population in a region were infected by AIDS-HIV. A sample of 200 people from this region were examined for AIDS-HIV. What was the expected number of infected people?

Chapter 5

Continuous Random Variables

5.1   Probability Density Function (pdf)

None

5.2   The Normal Random Variable

Use TI-84 "Normalcdf" function, under DISTR tab.

1. Suppose Z is the standard normal random variable. Find the probability $P(-1.23 \leq Z)$.

   Solution: $P(-1.23 \leq Z) = normalcdf(-1.23, 5) = .8907$

2. Suppose Z is the standard normal random variable. Find the probability $P(-1.18 \leq Z \leq 1.87)$.

   Solution: $P(-1.18 \leq Z \leq 1.87) = nomalcdf(-1.18, 1.87) = .8501$

3. Suppose X is a normal random variable with mean $\mu = 11.5$ and standard deviation $\sigma = 2.3$. Find the probability $P(X \leq 14)$.

Solution:

$P(X \leq 14) = P\left(\frac{X-\mu}{\sigma} \leq \frac{14-\mu}{\sigma}\right)$

$= P\left(Z \leq \frac{14-11.5}{2.3}\right) = P(Z \leq 1.0870) = nomalcdf(-5, 1.0870) = .8615$

4. Monthly consumption of electricity X (in KWH, in a winter month) by the households in a county has a normal distribution with mean 875 KWH and standard deviation 155 KWH. What proportions of households consumes less than 1000 KWH?

Solution:

$P(X \leq 1000) = P\left(\frac{X-\mu}{\sigma} \leq \frac{1000-\mu}{\sigma}\right)$

$= P\left(Z \leq \frac{1000-875}{155}\right) = P(Z \leq .8065) = nomalcdf(-5, 1.0870) = .7900$

5. The monthly cell phone minutes used by an individual in a city has normal distribution with mean 2700 minutes and standard deviation 325 minutes. What proportion of calls would last more than 3000 minutes?

Solution:

$P(3000 \leq X) = P\left(\frac{3000-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}\right)$

$= P\left(\frac{3000-2700}{325} \leq Z\right) = P(.9231 \leq Z) = normalcdf(.9231, 5) = .1780$

6. The amount of time a student spends to take a test has normal distributions with mean 45 minutes and standard deviation 13 minutes. What proportion of students would finish in 60 minutes?

Solution:

$P(X \leq 60) = P\left(\frac{X-\mu}{\sigma} \leq \frac{60-\mu}{\sigma}\right)$

$= P\left(Z \leq \frac{60-45}{13}\right) = P(Z \leq 1.1538) = nomalcdf(-5, 1.1538) = .8757$

7. The annual rainfall X in a region is normally distributed with mean $\mu = 62$ cm and standard deviation $\sigma = 9$ cm. What is the probability that rainfall will be between 55 cm and 70 cm?

Solution:

$P(55 \leq X \leq 70) = P\left(\frac{55-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{70-\mu}{\sigma}\right) = P\left(\frac{55-62}{9} \leq Z \leq \frac{70-62}{9}\right)$

$= P(-.7778 \leq Z \leq .8889) = nomalcdf(-.7778, .8889) = .5946$

8. The birth weight X of babies is normally distributed with mean $\mu = 113$ ounces and standard deviation $\sigma = 19$ ounces. What proportion of babies will have birth weight below 150 ounces?

Solution:

$P(X \leq 150) = P\left(\frac{X-\mu}{\sigma} \leq \frac{150-\mu}{\sigma}\right)$

$= P\left(Z \leq \frac{150-113}{19}\right) = P(Z \leq 1.9474) = nomalcdf(-5, 1.9474) = .9743$

9. The length X of babies at birth is normally distributed with mean $\mu = 18.5$ inches and standard deviation $\sigma = 2.3$ inch. What proportion of babies are smaller than 21 inches ?

   Solution:

   $P(X \leq 21) = P\left(\frac{X-\mu}{\sigma} \leq \frac{21-\mu}{\sigma}\right)$

   $= P\left(Z \leq \frac{21-18.5}{2.3}\right) = P(Z \leq 1.0870) = nomalcdf(-5, 1.0870) = .8615$

10. The weight X of salmon caught in a river is normally distributed with mean $\mu = 24$ pounds and standard deviation $\sigma = 7$ pounds. You can keep only those above 15 pounds. What proportion of the salmon caught can be kept?

    Solution:

    $P(15 \leq X) = P\left(\frac{15-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}\right)$

    $= P\left(\frac{15-24}{7} \leq Z\right) = P(-1.4286 \leq Z) = normalcdf(-1.4286, 5) = .9234$

5.2.1   Inverse Normal

Use TI-84 "invNorm" function, under DISTR tab.

1. Suppose Z is the standard normal random variable. It is given that $P(Z \leq c) = 0.2300$. What is the value of c?

   Solution:
   Answer $= invNorm(.23) = -.7388$

2. Suppose Z is the standard normal random variable. It is given that $P(d \leq Z) = 0.9500$. What is the value of d?

   Solution:
   $P(d < Z) = .95$
   $P(Z \leq d) = .05$
   Answer $= invNorm(.05) = -1.6449$

3. Suppose X is a normal random variable with mean $\mu = 3.5$ pounds and standard deviation $\sigma = 1.2$ pounds. Given that the probability $P(c \le X) = 0.2550$, what is the value of c? **Solution:** $\mu = 3.5, \sigma = 1.2$

$P(c < X) = 0.2550$
$P(X < c) = 1 - .2550 = .745$

$$\begin{cases} P\left(Z < \frac{c-\mu}{\sigma}\right) = .745 & \text{(This step iscalled Standardization)} \\ P(Z < .6588) = .745 & \text{Because } invNorm(.745) = .6588 \end{cases}$$

Compare:

$$\frac{c-\mu}{\sigma} = \frac{c-3.5}{1.2} = .6588 \Longrightarrow c = 3.5 + 1.2*.6588 = 4.2906$$

4. The weight X of babies (of a fixed age) is normally distributed with mean $\mu = 212$ ounces and standard deviation $\sigma = 25$ ounces. Doctors would be concerned (not necessarily alarmed) if a baby is among the lower 5 percent in weight. Find the cut-off weight l, below which the doctors will be concerned.

Solution: $\mu = 212, \sigma = 25,$
$P(X < c) = 0.05$

$$\begin{cases} P\left(Z < \frac{c-\mu}{\sigma}\right) = 0.05 & \text{(This step iscalled Standardization)} \\ P(Z < -1.6449) = 0.05 & \text{Because } invNorm(.05) = -1.6449 \end{cases}$$

Compare:

$$\frac{c-\mu}{\sigma} = \frac{c-212}{25} = -1.6449 \Longrightarrow c = 212 - 25*.1.6449 = 170.8775$$

5. The weight X of babies (of a fixed age) is normally distributed with mean $\mu = 212$ ounces and standard deviation $\sigma = 25$ ounces. Doctors would also be concerned (not necessarily alarmed) if a baby is among the upper 10 percent in weight. Find the cut-off weight u, above which the doctors will be concerned.

Solution: $\mu = 212, \sigma = 25,$
$P(u < X) = 0.10.$ So, $P(X < u) = .90$

$$\begin{cases} P\left(Z < \frac{u-\mu}{\sigma}\right) = .90 & \text{(This step iscalled Standardization)} \\ P(Z < 1.2816) = .90 & \text{Because } invNorm(.90) = 1.2816 \end{cases}$$

Compare:

$$\frac{u - \mu}{\sigma} = \frac{u - 212}{25} = 1.2816 \Longrightarrow u = 212 + 25 * 1.2816 = 244.04$$

6. The weight X of salmon caught in a river is normally distributed with mean $\mu = 24$ pounds and standard deviation $\sigma = 6$ pounds. You can keep those that are among that upper 66 percent in weight. What is the cut-off weight l, above which you can keep the fish?

Solution: $\mu = 24, \sigma = 6$,
$P(l < X) = .66.$ So, $P(X < l) = 1 - .66 = .34$

$$\begin{cases} P\left(Z < \frac{i-\mu}{\sigma}\right) = .34 & \text{(This step iscalled Standardization)} \\ P(Z < -.4125) = .34 & \text{Because} \quad invNoem(.34) = -.4125 \end{cases}$$

Compare:

$$\frac{l - \mu}{\sigma} = \frac{l - 24}{6} = -.4125 \Longrightarrow l = 24 - .4125 * 6 = 21.525$$

7. The height of an adult male in a region is known to be normally distributed with mean of $\mu = 69$ inches and a standard deviation $\sigma = 2.5$ inches. How high should a doorway be so that 92 percent of adult males can pass through it without having to bend?

Solution: $\mu = 69, \sigma = 2.5$,
$P(X < u) = .92$

$$\begin{cases} P\left(Z < \frac{u-\mu}{\sigma}\right) = .92 & \text{(This step iscalled Standardization)} \\ P(Z < 1.4051) = .92 & \text{Because} \quad invNoem(.92) = 1.4051 \end{cases}$$

Compare:

$$\frac{u - \mu}{\sigma} = \frac{u - 69}{2.5} = 1.4051 \Longrightarrow u = 69 + 1.4051 * 2.5 = 72.5128$$

8. The hourly wages X in an industry has a normal distribution with mean \$40 and standard deviation \$17. What is the 97 percentile of the hourly wage?

Solution: $\mu = 40, \sigma = 17,$
$P(X < u) = .97$

$$\begin{cases} P\left(Z < \frac{u-\mu}{\sigma}\right) = \ .97 & \text{(This step iscalled Standardization)} \\ P(Z < 1.8808) = .97 & \text{Because} \quad invNoem(.97) = 1.8808 \end{cases}$$

Compare:

$$\frac{u - \mu}{\sigma} = \frac{u - 40}{17} = 1.8808 \implies u = 40 + 1.8808 * 17 = 71.9736$$

9. The mean monthly water consumption X by the households in a subdivision has a normal distribution with mean 4500 gallons and standard deviation 3000 gallons. A surcharge would be imposed to the top 20 percent users. What is the cut-off number of gallons?

    Solution: $\mu = 4500, \sigma = 3000,$
    $P(u < X) = .20.$ So, $P(X < u) = .80$

    $$\begin{cases} P\left(Z < \frac{u-\mu}{\sigma}\right) = \ 80 & \text{(This step iscalled Standardization)} \\ P(Z < .8416) = .80 & \text{Because} \quad invNoem(.80) = .8416 \end{cases}$$

    Compare:

    $$\frac{u - \mu}{\sigma} = \frac{u - 4500}{3000} = .8416 \implies u = 4500 + \ .8416 * 3000 = 7024.8$$

10. The time X needed for a student to arrive at the class from his/her residence has a normal distribution with mean 30 minutes and with standard deviation 7 minutes. How much time before the class he/she should start so that he/she would be late only for 7 percent of the first period?

    Solution: $\mu = 30, \sigma = 7,$
    $P(u < X) = .07.$ So, $P(X < u) = .93$

    $$\begin{cases} P\left(Z < \frac{u-\mu}{\sigma}\right) = \ .93 & \text{(This step iscalled Standardization)} \\ P(Z < 1.4760) = .93 & \text{Because} \quad invNoem(.93) = 1.4760 \end{cases}$$

    Compare:

    $$\frac{u - \mu}{\sigma} = \frac{u - 30}{7} = 1.4760 \implies u = 30 + \ 7 * 1.4760 = 40332$$

5.3 Normal Approximation to Binomial

1. These are problems from Sec 5.3 Normal approximation to binomial.
   $n = 600$, $p = .40$
   mean $\mu = np = 600 * .4 = 240$
   $\sigma = \sqrt{np(1-p)} = \sqrt{600 * .4 * .6} = 12$
   $= P(220 < X) = P(221 \leq X) = P(220.5 < X)$
   (continuity correction.)
   $= P\left(\frac{220.5 - \mu}{\sigma}) < Z\right) = P\left(\frac{220.5 - 240}{\sqrt{(600 * .4 * .6)}}) < Z\right)$
   $= P(-1.625 < Z) = normalcdf(-1.625, 5) = .9479$

2. This is a problem from Sec 5.3 Normal approximation to binomial.
   $p = .42$, $n = 800$,
   $\mu = np = 800 * .42 = 336$
   $\sigma = \sqrt{np(1-p)} = \sqrt{800 * .42 * 58} = 13.9599$

   $P(X \leq 360) = P(X < 360.5)$
   (continuity correction.)
   $= P\left(Z < \frac{360.5 - \mu}{\sigma}\right) = P\left(Z < \frac{360.5 - 336}{13.9599}\right)$
   $= P(Z < 1.7550) = normalcdf(-5, 1.7550) = .9604$

3. This is a problem from Sec 5.3 Normal approximation to binomial.
   $p = .2$, $n = 400$
   $\mu = np = 400 * .2 = 80$
   $\sigma = \sqrt{np(1-p)} = \sqrt{400 * .2 * .8} = 8$
   $P(X < 74) = P(X \leq 73) = P(X < 73.5)$
   (continuity correction.)
   $= P\left(Z < \frac{73.5 - \mu}{\sigma}\right) = P\left(Z < \frac{73.5 - 80}{8}\right) = P(Z < [73.5 - 80]/8)$
   $= P(Z < -.8125) = normalcdf(-5, -.8125) = 0.2083$

4. These are problems from Sec 5.3 Normal approximation to binomial.
   $n = 256$, $p = .20$
   $\mu = np = 256 * .2 = 51.2$
   $\sigma_{\overline{X}} = \sqrt{np(1-p)} = \sqrt{256 * .2 * .8}$
   $P(60 < X) = P(61 \leq X) = P(60.5 < X)$
   (continuity correction.)
   $= P\left(\frac{60.5 - \mu}{\sigma} < Z\right) = P\left(\frac{60.5 - 51.2}{\sqrt{256 * .2 * .8}} < Z\right)$

$$= P(1.4531 < Z) = normalcdf(1.4531, 5) = .0731$$

5. Here $p = .15, \quad n = 60$

   $\mu = np = 60 * .15 = 9$

   $\sigma = \sqrt{np(1-p)} = \sqrt{60 * .15 * .85} = 2.7659$

   $P(X \le 10) = P(X < 10.5) = P\left((Z < \frac{10.5 - \mu}{\sigma}\right) = P\left(Z < \frac{10.5 - 9}{2.7659}\right)$

   $= P(Z < .5423) = normalcdf((-5, .5423) = .7062$

6. Here $p = .6$, $n = 150$.

   $\mu = np = 150 * .6 = 90$

   $\sigma = \sqrt{np(1-p)} = \sqrt{150 * .60 * .40} = 6$

   $P(X < 80) = P(X \le 79) = P(X < 79.5) = P\left(Z < \frac{79.5 - \mu}{\sigma}\right) = P\left(Z < \frac{79.5 - 90}{6}\right)$

   $= P(Z < -1.75) = normalcdf(-5, -1.75) = 0.0401$

7. Here $p = .55$, $n = 780$

   $\mu = np = 780 * .55 = 429$

   $\sigma = \sqrt{np(1-p)} = \sqrt{780 * .55 * .45} = 13.8942$

   $P(450 < X) = P(451 \le X) = P(450.5 < X) = P\left(\frac{450.5 - \mu}{\sigma} < Z\right) = P\left(\frac{450.5 - 429}{13.8942} < Z\right)$

   $= P(1.5474 < Z) = normalcdf(1.5474, 5) = .0609$

8. Here $p = .22$, $n = 600$

   $\mu = np = 600 * .22 = 132$

   $\sigma = \sqrt{np(1-p)} = \sqrt{600 * ..22 * .78} = 10.1469$

   $P(130 < X) = P(131 \le X) = P(130.5 < X)$

   $= P\left(\frac{130.5 - \mu}{\sigma} < Z\right) = P\left(\frac{130.5 - 132}{10.1469} < Z\right)$

   $= P(-.1478 < Z) = normalcdf(-.1478, 5) = .5587$

9. Here $p = .73$, $n = 400$.

   $\mu = np = 400 * .73 = 292$

   $\sigma = \sqrt{np(1-p)} = \sqrt{400 * 73 * .27} = 8.8792$

   $P(X < 300) = P(X \le 299) = P(X < 299.5)$

   $= P\left(Z < \frac{299.5 - \mu}{\sigma}\right) = P\left(Z < \frac{299.5 - 292}{8.8792}\right)$

   $= P(Z < .8447) = normalcdf(-5, .8447) = .8009$

10. Here $p = .25$, $n = 220$.

    $\mu = np = 220 * .25 = 55$

    $\sigma = \sqrt{np(1-p)} = \sqrt{220 * .25 * .75} = 6.4226$

$P(X < 50) = P(X \leq 49) = P(X < 49.5)$
$= P\left(Z < \frac{49.5-\mu]}{\sigma}\right) = P\left(Z < \frac{49.5-55}{6.4226}\right)$
$= P(Z < -8564) = normalcdf(-5, -8564) = .1959$

Chapter 6

Elements of Sampling Distribution

6.1 Sampling Distribution

None

6.2 Central Limit Theorem

1. We are asked to compute $P(1600 < \overline{X})$.
   The sample size $n = 64$. we use mean of $\mu = 1550$, and
   $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{416}{\sqrt{64}} = 52$
   So, $P(1600 < \overline{X}) = P\left(\frac{1600-\mu}{\sigma_{\overline{X}}} < Z\right)$
   $= P\left(\frac{1600-1550}{52} < Z\right) = P(.9615 < Z)$
   $= normalcdf(.9615, 5) = .1682$

2. We compute $P(\overline{X} < 230)$.

   $\mu = 225$ $\sigma = 45$, $n = 68$
   we use mean of $\mu = 225$, and

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{45}{\sqrt{68}} = 5.4571$

So, $P(\overline{X} < 230) = P\left(Z < \frac{230-\mu}{\sigma_{\overline{X}}}\right)$

$= P\left(Z < \frac{230-225}{5.4571}\right)$

$= P(Z < .9162) = normalcdf(-5, .9162) = .8202$

3. We compute $P(\overline{X} < 116)$.

$\mu = 114,\ \sigma = 18,\ n = 100$

Use mean of $\mu = 114$ and

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{100}} = 1.8$

So, $P(\overline{X} < 116) = P\left(Z < \frac{116-\mu)}{\sigma_{\overline{X}}}\right)$

$= P\left(Z < \frac{116-114}{1.8}\right) = P(Z < 1.1111)$

$= normalcdf(-5, 1.1111) = .8667$

4. We compute $P(\overline{X} < 36)$.

$\mu = 33,\ \sigma = 12,\ n = 35$

Use mean of $\mu = 33$ and

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{35}} = 2.0284$

So, $P(\overline{X} < 36) = P\left(Z < \frac{36-\mu)}{\sigma_{\overline{X}}}\right) = P\left(Z < \frac{36-33}{2.0284}\right)$

$= P(Z < 1.4790) = normalcdf(-5, 1.4790) = .9304$

5. We are asked to compute $P(50 < \overline{X})$.

Here, $n = 77,\ \mu = 48,\ \sigma = 19$

Use mean of $\mu = 48$, and

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{19}{\sqrt{77}} = 2.1653$

So, $P(50 < \overline{X}) = P\left(\frac{50-\mu)}{\sigma_{\overline{X}}} < Z\right) = P\left(\frac{50-48)}{2.1653} < Z\right)$

$= P(.9237 < Z) = normalcdf(.9237, 5) = .1778$

6. This question is posed in terms of 'Total'.

Note sample mean $= \overline{X} = \frac{Total}{n}$.

Question is $P(Total < 3000) = ?$

We change the problem to:

$P(Total < 3000) = P\left(\left(\frac{Total}{n} < \frac{3000}{n}\right)\right)$

$= P(\overline{X} < 3000/98) = P(\overline{X} < 30.6122)$

$\mu = 28,\ \sigma = 18,\ n = 98$

we use mean of $\overline{X} = mean = 28$ and

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{98}} = 1.8183$

So, $P(\overline{X} < 30.6122) = P\left(Z < \frac{30.6122 - \mu)}{\sigma_{\overline{X}}}\right)$

$= P\left(Z < \frac{30.6122 - 28)}{\sigma_{\overline{X}}}\right)$

$= P(Z < [30.6122 - 28]/1.8183) = P(Z < 1.4366)$

$= normalcdf(-5, 1.4366) = .9246$

7. This question is posed in terms of 'Total'.
   Note sample mean $= \overline{X} = \frac{Total}{n}$.
   Question is $P(900 < Total) =?$
   We change the problem to:
   $P(900 < Total) = P\left(\frac{900}{n} < \frac{Total}{n}\right)$
   $= P\left(\frac{900}{36} < \overline{X}\right) = P(25 < \overline{X})$

   $\mu = 24$, $\sigma = 8$, $n = 36$
   Use mean of $\mu = 24$ and
   $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{36} = 1.3333$
   So, $P(25 < \overline{X}) = P\left(\frac{25 - \mu)}{\sigma_{\overline{X}}} < Z\right)$
   $= P\left(\frac{25 - 24)}{1.3333} < Z\right) = P(.7500 < Z)$
   $= normalcdf(.7500, 5) = .2266$

8. This question is posed in terms of 'Total'.
   Note sample mean $= \overline{X} = \frac{Total}{n}$.
   Question is $P(130 < Total) =?$
   We change the problem to:

   $P(130 < Total) = P\left(\frac{130}{n} < \frac{Total}{n}\right) = P\left(10.8333 < \overline{X}\right)$
   $\mu = 10$, $\sigma = 4.4$, $n = 12$

   Use mean $\mu = 10$ and
   $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{4.4}{12} = 1.2702$
   So, $P(10.8333 < \overline{X}) = P\left(\frac{10.8333 - \mu}{\sigma_{\overline{X}}} < Z\right)$
   $= P\left(\frac{10.8333 - 10}{1.2702]} < Z\right) = P(.6560 < Z)$
   $= normalcdf(.6560, 5) = .2559$

9. This question is posed in terms of 'Total'.
   Note sample mean $= \overline{X} = \frac{Total}{n}$.
   Question is $P(Total < 1800) =?$

We change the problem to:

$P(Total < 1800) = P\left(\frac{Total}{n} < \frac{1800}{n}\right) == P\left(\frac{Total}{n} < \frac{1800}{33}\right) P\left(\overline{X} < 54.5455\right)$
Here $\mu = 55$, $\sigma = 22$, $n = 33$
Use $\mu = 55$ and
$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{22}{33} = 3.8297$
So, $P(\overline{X} < 54.5455)P\left(Z < \frac{54.5455-\mu}{\sigma_{\overline{X}}}\right) = P\left(Z < \frac{54.5455-55}{3.8297}\right)$
$= P(Z < -.1187) = normalcdf(-5, -.1187) = .4528$

10. This question is posed in terms of 'Total'.
    Note sample mean $= \overline{X} = \frac{Total}{n}$.
    Question is $P(6000 < Total) = ?$
    We change the problem to:

    $P(6000 < Total) = P\left(\frac{6000}{30} < \frac{Total}{n}\right) = P\left(200 < \overline{X}\right)$
    Here $\mu = 220$, $\sigma = 100$, $n = 30$
    Use $\mu = 220$ and
    $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{100}{30} = 18.2574$
    So, $P(200 < \overline{X}) = P\left(\frac{200-\mu}{\sigma_{\overline{X}}} < Z\right)$
    $= P\left(\frac{200-220}{18.2574} < Z\right) = P(-1.0954 < Z)$
    $= normalcdf(-1.0954, 5) = .8633$

6.3   Distribution of the Sample Proportion

1. These are problems on Sample Proportion to be solved using CLT.
   Write $X =$ Number of success.
   We are asked to find $P(400 < X)$.
   Here sample size $n = 680$, $p = .58$
   $\sigma_{\overline{X}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.58*.42}{680}}$
   $P(400 < X) = P\left(\frac{400}{n} < \frac{X}{n}\right) = P\left(\frac{400}{680} < \overline{X}\right)$
   $= P\left(\frac{400/680-p}{\sigma_{\overline{X}}} < Z\right) = P\left(\frac{400/680-.58}{\sqrt{.58*.42/680}} < Z\right)$
   $= P(.4351 < Z) = normalcdf(.4351, 5) = .3317$

2. These are problems on Sample Proportion to be solved using CLT.

Write X = Number of success. We are asked to find $P(X < 300)$.
Here sample size $n = 480$, $p = .65$

$\sigma_{\overline{X}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{.65 * .35/480}$

$P(X < 300) = P\left(\frac{X}{n} < \frac{300}{n}\right) = P\left(\overline{X} < \frac{300}{480}\right)$

$= P\left(Z < \frac{\frac{300}{480} - p}{\sigma_{\overline{X}}}\right) = P\left(Z < \frac{\frac{300}{480} - .65}{\sigma_{\overline{X}}}\right)$

$= P(Z < -1.1483) = normalcdf(-5, -1.1483) = .1254$

3. These are problems on Sample Proportion to be solved using CLT.
   Write X = Number of success. We are asked to find $P(X < 150)$.
   Here sample size $n = 200$, $p = .72$

   $\sigma_{\overline{X}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{.72 * .28/200}$

   $P(X < 150) = P\left(\frac{X}{n} < 1\frac{150}{n}\right) = P\left(\overline{X} < \frac{150}{200}\right)$

   $= P\left(Z < \frac{\frac{150}{200} - p}{\sigma_{\overline{X}}}\right) = P\left(Z < \frac{\frac{150}{200} - .72}{\sqrt{(.72*.28/200)}}\right)$

   $= P(Z < .9449) = normalcdf(-5, .9449) = .8276$

4. These are problems on Sample Proportion to be solved using CLT.
   Write X = Number of success. We are asked to find $P(370 < X)$.
   Here sample size $n = 720$, $p = .52$

   $\sigma_{\overline{X}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{.52 * .48/720}$

   $P(370 < X) = P\left(\frac{370}{n} < \frac{X}{n}\right) = P\left(400/720 < \overline{X}\right)$

   $= P\left(\frac{\frac{370}{720} - p}{\sigma_{\overline{X}}} < Z\right) = P\left(\frac{\frac{370}{720} - .52}{\sqrt{.52*.48/720}} < Z\right)$

   $= P(.3282 < Z) = normalcdf(-.3282, 5) = .6286$

5. These are problems on Sample Proportion to be solved using CLT.
   Write X = Number of success. We are asked to find $P(100 < X)$.
   Here sample size $n = 120$, $p = .80$

   $\sigma_{\overline{X}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{.80 * .20/120}$

   $P(100 < X) = P\left(\frac{100}{n} < \frac{X}{n}\right) = P\left(\frac{100}{120} < \overline{X}\right)$

   $= P\left(\frac{\frac{100}{120} - p}{\sigma_{\overline{X}}} < Z\right) = P\left(\frac{\frac{100}{120} - .80}{\sigma_{\overline{X}}} < Z\right)$

   $= P(.9128 < Z) = normalcdf(.9128, 5) = .1807$

6. These are problems on Sample Proportion to be solved using CLT.
   Write X = Number of success. We are asked to find $P(X < 100)$.

Here sample size $n = 250$, $p = .43$

$\sigma_{\overline{X}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{.43 * .57/250}$

$P(X < 100) = P\left(\frac{X}{n} < \frac{100}{n}\right) = P\left(\overline{X} < \frac{100}{250}\right)$

$= P\left(Z < \frac{\frac{100}{250} - p}{\sigma_{\overline{X}}}\right) = P\left(Z < \frac{\frac{100}{250} - .43}{\sqrt{.43*.57/250}}\right)$

$= P(Z < -.9581) = normalcdf(-5, -.9581) = .1690$

7. These are problems on Sample Proportion to be solved using CLT.
   Write $X = $ Number of success. We are asked to find $P(X < 215)$.
   Here sample size $n = 800$, $p = .25$

   $\sigma_{\overline{X}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{.25 * .75/800}$

   $P(X < 215) = P\left(\frac{X}{n} < \frac{215}{n}\right) = P\left(\overline{X} < \frac{215}{800}\right)$

   $= P\left(Z < \frac{\frac{215}{800} - p}{\sigma_{\overline{X}}}\right) = P\left(Z < \frac{\frac{215}{800} - .25}{\sqrt{.25*.75/800}}\right)$

   $= P(Z < 1.2247) = normalcdf(-5, 1.2247) = .8896$

8. These are problems on Sample Proportion to be solved using CLT.
   Write $X = $ Number of success. We are asked to find $P(X < 100)$.
   Here sample size $n = 700$, $p = .15$

   $\sigma_{\overline{X}} = \sqrt{\frac{p(1-p)}{n}} == \sqrt{.15 * .85/700}$

   $P(X < 100) = P\left(\frac{X}{n} < \frac{100}{n}\right) = P\left(\overline{X} < \frac{100}{700}\right)$

   $= P\left(Z < \frac{\frac{100}{700} - p}{\sigma_{\overline{X}}}\right) = P\left(Z < \frac{\frac{100}{700} - .15}{\sqrt{.15*.85/700}}\right)$

   $= P(Z < -.5293) = normalcdf(-5, -.5293) = .2983$

Chapter 7

Estimation

7.1 Point and Interval Estimation

7.1.1 Confidence Interval of μ

1. Here the population standard deviation $\sigma = 10$
   the sample size $n = 49$
   the sample mean $\overline{X} = 100$
   Level of confidence 99 percent. So,
   $1 - \alpha = .99$, $\alpha/2 = .005$. Therefore,
   $z_{\alpha/2} = z_{.005} = invNormal(1 - .01) = 2.5758$
   MOE$= \frac{z_{\alpha/2}\sigma}{\sqrt{n}} = \frac{2.5758*10}{\sqrt{49}} = 3.6797$
   LEP$= \overline{X} - E = 100 - 3.6797$
   LEP$= \overline{X} + E = 100 + 3.6797$

2. Here the population standard deviation $\sigma = 13.5$
   the sample size $n = 87$
   the sample mean $\overline{X} = 68$
   Level of confidence 96 percent. So,
   $1 - \alpha = .96$ $\alpha/2 = .02$. Therefore,
   $z_{\alpha/2} = z_{.02} = invNormal(1 - .02) = 2.0537$

MOE= $\frac{z_{\alpha/2}\sigma}{\sqrt{n}} = \frac{2.0537*13.5}{\sqrt{87}} = 2.9724$

LEP= $\overline{X} - E = 68 - 2.9724$

LEP= $\overline{X} + E = 68 + 2.9724$

3. Here the population standard deviation $\sigma = 7.5$

   the sample size $n = 116$

   the sample mean $\overline{X} = 22$

   Level of confidence 98 percent. So,

   $1 - \alpha = .98$ $\alpha/2 = .01$. Therefore,

   $z_{\alpha/2} = z_{.01} = invNormal(1 - .01) = 2.3263$

   MOE= $\frac{z_{\alpha/2}\sigma}{\sqrt{n}} = \frac{2.3263*7.5}{\sqrt{116}} = 1.6199$

   LEP= $\overline{X} - E = 22 - 1.6199$

   LEP= $\overline{X} + E = 22 - 1.6199$

4. Here the population standard deviation $\sigma = 3.5$

   the sample size $n = 30$

   the sample mean $\overline{X} = 25$

   Level of confidence 95 percent. So,

   $1 - \alpha = .95$ $\alpha/2 = .025$. Therefore,

   $z_{\alpha/2} = z_{.**} = invNormal(1 - .025) = 1.9600$

   MOE= $\frac{z_{\alpha/2}\sigma}{\sqrt{n}} = \frac{1.9600*3.5}{\sqrt{30}} = 1.2525$

   LEP= $\overline{X} - E = 25 - 1.2525$

   LEP= $\overline{X} + E = 25 + 1.2525$

5. Here the population standard deviation $\sigma = 1278$

   the sample size $n = 180$

   the sample mean $\overline{X} = 4200$

   Level of confidence 92 percent. So,

   $1 - \alpha = .92$ $\alpha/2 = .04$. Therefore,

   $z_{\alpha/2} = z_{.04} = invNormal(1 - .04) = 1.7507$

   MOE= $\frac{z_{\alpha/2}\sigma}{\sqrt{n}} = \frac{z_{1.7507*1278}}{\sqrt{180}} = 166.7655$

   LEP= $\overline{X} - E = 4200 - 166.7655$

   LEP= $\overline{X} + E = 4200 + 166.7655$

6. Here the population standard deviation $\sigma = 6$

   the sample size $n = 56$

   the sample mean $\overline{X} = 21.1$

Level of confidence 90 percent. So,
$1 - \alpha = .90 \ \alpha/2 = .05$. Therefore,
$z_{\alpha/2} = z_{.05} = invNormal(1 - .05) =$
MOE= $\frac{z_{\alpha/2}\sigma}{\sqrt{n}} = \frac{z_{\alpha/2}\sigma}{\sqrt{n}} = 1.6449$
LEP= $\overline{X} - E = 21.1 - 1.6449$
LEP= $\overline{X} + E = 21.1 + 1.6449$

7.1.2 The Required Sample Size

1. Here the population standard deviation $\sigma = 45$
   and maximum error $E = 5$
   Level of confidence 99 percent. So
   $1 - \alpha = .99 \ \alpha/2 = .005$. Therefore,
   $z_{\alpha/2} = z_{.01} = invNormal(1 - .005) = 2.5758$
   The sample size $n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.5758*45}{5}\right)^2 = 537.41$
   We would round it upward. So, Answer $= 538$

2. Here the population standard deviation $\sigma = 20$
   and maximum error $E = 2$
   Level of confidence 98 percent. So
   $1 - \alpha = .98 \ \alpha/2 = .01$. Therefore,
   $z_{\alpha/2} = z_{.01} = invNormal(1 - .01) = 2.3263$
   The sample size $n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.3263*20}{2}\right)^2 = 541.17$
   We would round it upward. So, Answer $= 542$

3. Here the population standard deviation $\sigma = 13$
   and maximum error $E = 1$
   Level of confidence 96 percent. So
   $1 - \alpha = .96 \ \alpha/2 = .02$. Therefore,
   $z_{\alpha/2} = z_{.02} = invNormal(1 - .02) = 2.0537$
   The sample size $n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.0537*13}{1}\right)^2 = 712.79$
   We would round it upward. So, Answer $= 713$

4. Here the population standard deviation $\sigma = 87$
   and maximum error $E = 5$
   Level of confidence 94 percent. So
   $1 - \alpha = .94 \ \alpha/2 = .03$. Therefore,

$z_{\alpha/2} = z_{.03} = invNormal(1 - .03) = 1.8808$

The sample size $n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.8808*87}{5}\right)^2 = 1070.99$

We would round it upward. So, Answer $= 1071$

5. Here the population standard deviation $\sigma = 540$
   and maximum error $E = 100$
   Level of confidence 97 percent. So
   $1 - \alpha = .97$ $\alpha/2 = .015$. Therefore,
   $z_{\alpha/2} = z_{.015} = invNormal(1 - .015) = 2.1701$
   The sample size $n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.1701*540}{100}\right)^2 = 137.32$
   We would round it upward. So, Answer $= 138$

6. Here the population standard deviation $\sigma = 1.5$ and maximum error
   $E = .5$
   Level of confidence 80 percent. So
   $1 - \alpha = .80$ $\alpha/2 = .1$. Therefore,
   $z_{\alpha/2} = z_{.1} = invNormal(1 - .1) = 1.2816$
   The sample size $n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.2816*1.5}{.5}\right)^2 = 14.78$
   We would round it upward. So, Answer $= 25$

7.2   Confidence interval for μ when σ is unknown

These are also known as T-intervals.

1. Here the sample size $n = 22$
   the sample mean $\overline{X} = 155.1$
   Sample standard deviation $s = 14.9$
   The degrees of freedom $df = n - 1 = 22 - 1 = 21$
   Level of confidence: 95 percent. So
   $1 - \alpha = .95$ $\alpha/2 = .025$. Therefore,
   $t_{n-1,\alpha/2} = invT(1 - .025, 21) = 2.0796$
   MOE$= \frac{t_{n-1,\alpha/2}s}{\sqrt{n}} = \frac{2.0796*14.9}{\sqrt{22}} = 6.6062$
   LEP$= \overline{X} - E = 155.1 - 6.6062$
   LEP$= \overline{X} + E = 155.1 + 6.6062$

2. Here the sample size $n = 250$
the sample mean $\overline{X} = 129.1$
Sample standard deviation $s = 11.95$
The degrees of freedom $df = n - 1 = 250 - 1 = 249$
Level of confidence: 99 percent. So
$1 - \alpha = .99$ $\alpha/2 = .005$. Therefore,
$t_{n-1,\alpha/2} = invT(1 - .005, 249) = 2.5957$
MOE$= \frac{t_{n-1,\alpha/2}s}{\sqrt{n}} = \frac{2.5957*11.95}{\sqrt{249}} = 1.9657$
LEP$= \overline{X} - E = 129.1 - 1.9657$
LEP$= \overline{X} + E = 129.1 - 1.9657$

3. Here the sample size $n = 131$
the sample mean $\overline{X} = 101.6$
Sample standard deviation $s = 11.1$
The degrees of freedom $df = n - 1 = 131 - 1 = 130$
Level of confidence: 96 percent. So
$1 - \alpha = .96$ $\alpha/2 = .02$. Therefore,
$t_{n-1,\alpha/2} = invT(1 - .02, 130) = 2.0746$
MOE$= \frac{t_{n-1,\alpha/2}s}{\sqrt{n}} = \frac{2.0746*11.1}{\sqrt{131}} = 2.0120$
LEP$= \overline{X} - E = 101.6 - 2.0120$
LEP$= \overline{X} + E = 101.6 + 2.0120$

4. I used TI-84 to summarize the data:
the sample size $n = 18$
the sample mean $\overline{X} = 4145.11$
Sample standard deviation $s = 2219.86$
The degrees of freedom $df = n - 1 = 18 - 1 = 17$
Level of confidence: ?? percent. So
$1 - \alpha = .95$ $\alpha/2 = .025$. Therefore,
$t_{n-1,\alpha/2} = invT(1 - .025, 17) = 2.1098$
MOE$= \frac{t_{n-1,\alpha/2}s}{\sqrt{n}} = \frac{2.1098*2219.86}{\sqrt{18}} = 1103.90$
LEP$= \overline{X} - E = 4145.11 - 1103.90$
LEP$= \overline{X} + E = 4145.11 + 1103.90$

5. I used TI-84 to summarize the data:
the sample size $n = 19$
the sample mean $\overline{X} = 51.3895$
Sample standard deviation $s = 2.6143$

The degrees of freedom $df = n - 1 = 19 - 1 = 18$
Level of confidence: 95 percent. So
$1 - \alpha = .95$ $\alpha/2 = .025$. Therefore,
$t_{n-1,\alpha/2} = invT(1 - .025, 18) = 2.1009$
MOE= $\frac{t_{n-1,\alpha/2}s}{\sqrt{n}} = \frac{2.1009*2.6143}{\sqrt{19}} = 1.2600$
LEP= $\overline{X} - E = 51.3895 - 1.2600$
LEP= $\overline{X} + E = 51.3895 + 1.2600$

6. Here the sample size $n = 19$
   the sample mean $\overline{X} = 1.4684$
   Sample standard deviation $s = .1924$
   The degrees of freedom $df = n - 1 = 19 - 1 = 18$
   Level of confidence: 99 percent. So
   $1 - \alpha = .99$ $\alpha/2 = .005$. Therefore,
   $t_{n-1,\alpha/2} = invT(1 - .005, 18) = 2.8784$
   MOE= $\frac{t_{n-1,\alpha/2}s}{\sqrt{n}} = \frac{2.8784*.1924}{\sqrt{19}} = .1271$
   LEP= $\overline{X} - E = 1.4684 - .1271$
   LEP= $\overline{X} + E = 1.4684 + .1271$

7.3   About the Population Proportion

1. Here sample size $n = 180$
   The number of success $T = 27$
   So, the sample proportion of success $\overline{X} = \frac{T}{n} = \frac{27}{180} = .15$
   Given level of confidence is 95 percent.
   $1 - \alpha = .95$, $\alpha/2 = .025$, $z_{\alpha/2} = z_{.025} = invNormal(1 - .025) = 1.9600$
   MOE= $e = z_{\alpha/2}\sqrt{\frac{\overline{X}(1-\overline{X})}{n}} = 1.9600 * \sqrt{\frac{.15(1-.15)}{180}} = .052164$
   LEP= $\overline{X} - e = .15 - .052164$
   LEP= $\overline{X} + e = .15 + .052164$
   Conservative MOE $= E = \frac{z_{\alpha/2}}{\sqrt{4n}} = \frac{1.9600}{\sqrt{4*180}} = .073304$
   (In this section, for error terms, we retain at least 6 decimal points.)

2. Here sample size $n = 176$
   The number of success $T = 44$

So, the sample proportion of success $\overline{X} = \frac{T}{n} = \frac{44}{176} = .25$
Given level of confidence is 99 percent.
$1 - \alpha = .99$, $\alpha/2 = .005$, $z_{\alpha/2} = z_{.005} = invNormal(1 - .005) = 2.5758$

MOE$= e = z_{\alpha/2}\sqrt{\frac{\overline{X}(1-\overline{X})}{n}} = 2.5758 * \sqrt{\frac{.25(1-.25)}{176}} = .084073$

LEP$= \overline{X} - e = .25 - .084073$
LEP$= \overline{X} + e = .25 - .084073$
Conservative MOE $= E = \frac{z_{\alpha/2}}{\sqrt{4n}} = \frac{2.5758}{\sqrt{4*176}} = .097079$
(In this section, for error terms, we retain at least 6 decimal points.)

3. Here given precision $E = .01$
   Given level of confidence is 95 percent.
   $1 - \alpha = .95$, $\alpha/2 = .025$, $z_{\alpha/2} = z_{.025} = invNormal(1 - .025) = 1.9600$
   The required sample size $n = \left(\frac{z_{\alpha/2}}{2E}\right)^2 = \left(\frac{1.9600}{2*.01}\right)^2 = 9604$

4. Here given precision $E = .02$
   Given level of confidence is 99 percent.
   $1 - \alpha = .99$, $\alpha/2 = .005$, $z_{\alpha/2} = z_{.005} = invNormal(1 - .005) = 2.5758$
   The required sample size $n = \left(\frac{z_{\alpha/2}}{2E}\right)^2 = \left(\frac{2.5758}{2*.02}\right)^2 = 4146.7160$ We would
   round it upward. So,
   Answer $= 4147$

7.4 Confidence Interval of the Variance σ^2

1. Use TI to compute
   the sample size $n = 28$
   the sample variance $s^2 2 = (12.3135)^2 = 151.6223$
   Degrees of freedom $df = n - 1 = 27$
   Given level of confidence is 95 percent.
   $1 - \alpha = .95$, $\alpha/2 = .025$, Using the Inverse Chi-Square animation 7.4.1
   we have
   $\chi^2_{n-1,\alpha/2} = \chi^2_{27,.025} = 43.195$
   $\chi^2_{n-1,1-\alpha/2} = \chi^2_{27,1-.025} = 14.574$
   Therefore, LEP$= \frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} = \frac{27*151.6223}{43.195}$

$$\text{REP} = \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}} = \frac{27*151.6223}{14.574}$$

2. the sample size $n = 16$
   the sample variance $s^2 = 313$
   Degrees of freedom $df = n - 1 = 17$
   Given level of confidence is 99 percent.
   $1 - \alpha = .99$, $\alpha/2 = .005$, Using the Inverse Chi-Square animation 7.4.1
   we have
   $\chi^2_{n-1,\alpha/2} = \chi^2_{17,.005} = 35.719$
   $\chi^2_{n-1,1-\alpha/2} = \chi^2_{17,1-.005} = 5.698$
   Therefore, LEP$= \frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} = \frac{17*313}{5.698}$
   REP $= \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}} = \frac{17*313}{5.698}$

3. Use TI to compute
   the sample size $n = 20$
   the sample variance $s^2 2 = (1.4162)^2 = 2.0056$
   Degrees of freedom $df = n - 1 = 19$
   Given level of confidence is 90 percent.
   $1 - \alpha = .90$, $\alpha/2 = .05$, Using the Inverse Chi-Square animation 7.4.1
   we have
   $\chi^2_{n-1,\alpha/2} = \chi^2_{19,.05} = 30.144$
   $\chi^2_{n-1,1-\alpha/2} = \chi^2_{19,1-.05} = 10.118$
   Therefore, LEP$= \frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} = \frac{19*2.0056}{30.144}$
   REP $= \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}} = \frac{19*2.0056}{10.118}$

Chapter 8

Comparing Two Populations

8.1 Confidence Interval of $\mu_1 - \mu_2$

1. The given data is summarizes as follows:

	Population I (X)	**Population II (Y)**
Sample Size	$m = 36$	$n = 34$
Population Mean	$\overline{X} = 60.6$	$\overline{Y} = 63.7$
Population St. Dev	$\sigma_1 = 11$	$\sigma_2 = 13$

Given level of confidence is 99 percent.

$1 - \alpha = .99$, $\alpha/2 = .005$, $z_{\alpha/2} = z_{.005} = invNormal(1 - .005) = 2.5758$

MOE= $z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}} = 2.5758\sqrt{\frac{11^2}{36} + \frac{13^2}{34}} = 7.4350$

So, LEP= $(\overline{X} - \overline{Y}) - E = (60.6 - 63.7) - 7.4350$

REP= $(\overline{X} - \overline{Y}) - E = (60.6 - 63.7) + 7.4350$

2. The given data is summarizes as follows:

	Population I (X)	**Population II (Y)**
Sample Size	$m = 40$	$n = 36$
Population Mean	$\overline{X} = 375$	$\overline{Y} = 365$
Population St. Dev	$\sigma_1 = 20$	$\sigma_2 = 10$

Given level of confidence is 95 percent.

$1 - \alpha = .95$, $\alpha/2 = .025$, $z_{\alpha/2} = z_{.025} = invNormal(1 - .025) = 1.9600$

MOE= $E = z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}$

$= 1.9600\sqrt{\frac{20^2}{40} + \frac{10^2}{36}} = 7.0062$

So, LEP= $(\overline{X} - \overline{Y}) - E = (375 - 365) - 7.0062$

REP= $(\overline{X} - \overline{Y}) - E = (375 - 365) + 7.0062$

3. The given data is summarizes as follows:

	Population I (X)	**Population II** (Y)
Sample Size	$m = 18$	$n = 32$
Population Mean	$\overline{X} = 11.2$	$\overline{Y} = 9.2$
Population St. Dev	$\sigma_1 = 1.5$	$\sigma_2 = 1$

Given level of confidence is 98 percent.

$1 - \alpha = .98$, $\alpha/2 = .01$, $z_{\alpha/2} = z_{.01} = invNormal(1 - .01) = 2.3263$

MOE= $E = z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}$

$= 2.3263\sqrt{\frac{1.5^2}{18} + \frac{1^2}{32}} = .9195$

So, LEP= $(\overline{X} - \overline{Y}) - E = (11.2 - 9.2) - .9195$

REP= $(\overline{X} - \overline{Y}) - E = (11.2 - 9.2) + .9195$

4. The given data is summarizes as follows:

	Population I (X)	**Population II** (Y)
Sample Size	$m = 21$	$n = 48$
Population Mean	$\overline{X} = 19.9$	$\overline{Y} = 17.4$
Population St. Dev	$\sigma_1 = 2.3$	$\sigma_2 = 2.9$

Given level of confidence is 97 percent.

$1 - \alpha = .97$, $\alpha/2 = .015$, $z_{\alpha/2} = z_{.015} = invNormal(1 - .015) = 2.1701$

MOE= $E = z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}$

$= 2.1701\sqrt{\frac{2.3^2}{21} + \frac{2.9^2}{48}} = 1.4182$

So, LEP= $(\overline{X} - \overline{Y}) - E = (19.9 - 17.4) - 1.4182$

REP= $(\overline{X} - \overline{Y}) - E = (19.9 - 17.4) + 1.4182$

8.2 When σ_1 and σ_2 are unknown

1. Use TI and summarize the data as follows:

	Population I (X)	Population II (Y)
Sample Size	$m = 20$	$n = 21$
Sample Mean	$\overline{X} = 10.815$	$\overline{Y} = 9.0190$
Sample St. Dev	$s_1 = 1.4162$	$s_2 = .8072$

The pooled estimate of σ is given by

$$S_p = \sqrt{\frac{(m-1)S_{\overline{X}}^2 + (n-1)S_{\overline{X}}^2}{m+n-2}} = \sqrt{\frac{19*(1.4162)^2 + 20*(.8072)^2}{39}} = 1.1451$$

Degrees of freedom
$df = m + n - 2 = 20 + 21 - 2 = 39$
Given level of confidence is 97 percent. Therefore
$1 - \alpha = .97$, $\alpha/2 = .015$,
$t_{n-1,\alpha/2} = invT(1 - .015, 39) = 2.2524$
MOE= $E = t_{m+n-2,\alpha/2} S_p \sqrt{\frac{1}{m} + \frac{1}{n}} = 2.2524 * 1.1451 \sqrt{\frac{1}{20} + \frac{1}{21}} = .8059$
So, LEP= $(\overline{X} - \overline{Y}) - E = (10.815 - 9.0190) - .8059$
REP= $(\overline{X} - \overline{Y}) - E = (10.815 - 9.0190) + .8059$

2. Summarize the data as follows:

	Population I (X)	Population II (Y)
Sample Size	$m = 13$	$n = 12$
Sample Mean	$\overline{X} = 19.9$	$\overline{Y} = 17.4$
Sample St. Dev	$s_1 = 1.8$	$s_2 = 2.2$

The pooled estimate of σ is given by

$$S_p = \sqrt{\frac{(m-1)S_{\overline{X}}^2 + (n-1)S_{\overline{X}}^2}{m+n-2}} = \sqrt{\frac{12*(1.8)^2 + 11*(2.2)^2}{23}} = 2.0013$$

Degrees of freedom
$df = m + n - 2 = 13 + 12 - 2 = 23$

Given level of confidence is 98 percent. Therefore
$1 - \alpha = .98$, $\alpha/2 = .01$,
$t_{n-1,\alpha/2} = invT(1 - .01, 23) = 2.4999$
MOE= $E = t_{m+n-2,\alpha/2}S_p\sqrt{\frac{1}{m} + \frac{1}{n}} = 2.4999 * 2.0013\sqrt{\frac{1}{13} + \frac{1}{12}} = 2.0028$
So, LEP= $(\overline{X} - \overline{Y}) - E = (19.9 - 17.4) - 2.0028$
REP= $(\overline{X} - \overline{Y}) - E = (19.9 - 17.4) + 2.0028$

3. Summarize the data as follows:

	Population I (X)	**Population II** (Y)
Sample Size	$m = 14$	$n = 11$
Sample Mean	$\overline{X} = 3.75$	$\overline{Y} = 3.65$
Sample St. Dev	$s_1 = 0.22$	$s_2 = 0.18.$

The pooled estimate of σ is given by

$$S_p = \sqrt{\frac{(m-1)S_{\overline{X}}^2 + (n-1)S_{\overline{X}}^2}{m+n-2}} = \sqrt{\frac{13 * (0.22)^2 + 10 * (0.18)^2}{23}} = .2036$$

Degrees of freedom
$df = m + n - 2 = 14 + 11 - 2 = 23$
Given level of confidence is 95 percent. Therefore
$1 - \alpha = .95$, $\alpha/2 = .025$,
$t_{n-1,\alpha/2} = invT(1 - .025, 23) = 2.0687$
MOE= $E = t_{m+n-2,\alpha/2}S_p\sqrt{\frac{1}{m} + \frac{1}{n}} = 2.0687 * .2036\sqrt{\frac{1}{14} + \frac{1}{11}} = .1697$
So, LEP= $(\overline{X} - \overline{Y}) - E = (3.75 - 3.65) - .1697$
REP= $(\overline{X} - \overline{Y}) - E = (3.75 - 3.65) + .1697$

8.3   Comparing Two Population Proportions

1. Summarize the data as follows:

	Population I (X)	**Population II** (Y)
Sample Size	$m = 227$	$n = 156$
NumberofSuccess	$X = 106$	$Y = 48$
Sample Prop.ofSuccess	$\overline{X} = \frac{X}{m} = .4670$	$\overline{Y} = \frac{Y}{n} = .3077$

Given level of confidence is 97 percent. Therefore
$1 - \alpha = .97$, $\alpha/2 = .015$,
$z_{\alpha/2} = invNorm(1 - .015) = 2.1701$
MOE= $E = z_{\alpha/2}\sqrt{\frac{\overline{X}(1-\overline{X})}{m} + \frac{\overline{Y}(1-\overline{Y})}{n}}$
$= 2.1701\sqrt{\frac{.4670(1-.4670)}{227} + \frac{.3077(1-.3077)}{156}} = .107678$
So, LEP= $(\overline{X} - \overline{Y}) - E = (.4670 - .3077) - .107678$
REP= $(\overline{X} - \overline{Y}) - E = (.4670 - .3077) + .107678$

2. Summarize the data as follows:

	Population I (X)	Population II (Y)
Sample Size	$m = 210$	$n = 270$
NumberofSuccess	$X = 97$	$Y = 88$
Sample Prop.ofSuccess	$\overline{X} = \frac{X}{m} = .4619$	$\overline{Y} = \frac{Y}{n} = .3259$

Given level of confidence is 99 percent. Therefore
$1 - \alpha = .99$, $\alpha/2 = .005$,
$z_{\alpha/2} = invNorm(1 - .005) = 2.5758$
MOE= $E = z_{\alpha/2}\sqrt{\frac{\overline{X}(1-\overline{X})}{m} + \frac{\overline{Y}(1-\overline{Y})}{n}}$
$= 2.5758\sqrt{\frac{.4619(1-.4619)}{210} + \frac{.3259(1-.3259)}{270}} = .115113$
So, LEP= $(\overline{X} - \overline{Y}) - E = (.4619 - .3259) - .115113$
REP= $(\overline{X} - \overline{Y}) - E = (.4619 - .3259) + .115113$

3. Summarize the data as follows:

	Population I (X)	Population II (Y)
Sample Size	$m = 270$	$n = 220$
NumberofSuccess	$X = 37$	$Y = 27$
Sample Prop.ofSuccess	$\overline{X} = \frac{X}{m} = .1370$	$\overline{Y} = \frac{Y}{n} = .1227$

Given level of confidence is 95 percent. Therefore
$1 - \alpha = .95$, $\alpha/2 = .025$,
$z_{\alpha/2} = invNorm(1 - .025) = 1.9600$
MOE= $E = z_{\alpha/2}\sqrt{\frac{\overline{X}(1-\overline{X})}{m} + \frac{\overline{Y}(1-\overline{Y})}{n}}$

$$= 1.9600\sqrt{\frac{.1370(1-.1370)}{270} + \frac{.1227(1-.1227)}{220}} = .059681$$

So, LEP$= (\overline{X} - \overline{Y}) - E = (.1370 - .1227) - .059681$

REP$= (\overline{X} - \overline{Y}) - E = (.1370 - .1227) + .059681$

Chapter 9

Testing Hypotheses

9.1 A Significance Test for mean μ when σ is known

These are also called Z-Tests.

1. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \mu = 51 \\ H_A : \mu > 51 \end{cases}$$

So, $\mu_0 = 51$, The sample size $n = 15$
The known population standard deviation $\sigma = 5$, Observed Sample mean $\overline{x} = 54$
$z = \frac{(\overline{x} - \mu_0)\sqrt{n}}{\sigma} = \frac{(54-51)\sqrt{15}}{5} = 2.3238$
This is a Right Tail Test. So,
p-value $p_v = P(z < Z) = normalcdf(z, 5)$
$= normalcdf(2.3238, 5) = .0101$

   (a) Given Level of Significance is 5 percent. So, $\alpha = .05$.
   We have $p_v = .0101 < \alpha = .05$. So, we reject H_0 at 5 percent level.

(b) We have $.01 < p_v = .0101 < .02$. So lowest level of rejection of H_0 is 2 percent.

2. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \mu = 112 \\ H_A : \mu > 112 \end{cases}$$

So, $\mu_0 = 112$, The sample size $n = 96$
The known population standard deviation $\sigma = 17$, Observed Sample mean $\overline{x} = 115$
$z = \frac{(\overline{x}-\mu_0)\sqrt{n}}{\sigma} = \frac{(115-112)\sqrt{96}}{17} = 1.7291$
This is a Right Tail Test. So,
p-value $p_v = P(z < Z) = normalcdf(z, 5)$
$= normalcdf(1.7291, 5) = .0419$

(a) Given Level of Significance is 7 percent. So, $\alpha = .07$.
We have $p_v = .0419 < \alpha = .07$. So, we reject H_0 at 7 percent level
.

(b) We have $.04 < p_v = .0419 < .05$. So lowest level of rejection of H_0 is 5 percent.

3. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \mu = 25 \\ H_A : \mu < 25 \end{cases}$$

So, $\mu_0 = 25$, The sample size $n = 29$
The known population standard deviation $\sigma = 3$, Observed Sample mean $\overline{x} = 23.9$
$z = \frac{(\overline{x}-\mu_0)\sqrt{n}}{\sigma} = \frac{(23.9-25)\sqrt{29}}{3} = -1.9746$
This is a Left Tail Test. So,
p-value $p_v = P(Z < z) = normalcdf(-5, z)$
$= normalcdf(-5, -1.9746) = .0242$.

(a) Given Level of Significance is 6 percent. So, $\alpha = .06$.
We have $p_v = .0242 < \alpha = .06$. So, we reject H_0 at 6 percent level
.

(b) We have $.02 < p_v = .0242 < .03$. So lowest level of rejection of H_0 is 3 percent.

9.2 A Significance Test for mean μ when σ is unknown

These are also called T-Tests.

1. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \mu = 23 \\ H_A : \mu < 23 \end{cases}$$

So, $\mu_0 = 23$, The sample size $n = 46$
Observed Sample mean $\overline{x} = 21.1$
Observed sample standard deviation $s = 6.$,
$t = \frac{(\overline{x}-\mu_0)\sqrt{n}}{s} = \frac{(21.1-23)\sqrt{46}}{6} = -2.1477$
This is a Left Tail Test. So,
p-value $p_v = P(T < t) = tcdf(-5, t, n-1)$
$= tcdf(-5, -2.1477, 45) = .0186.$

 (a) Given Level of Significance is 6 percent. So, $\alpha = .06$.
 We have $p_v = .0186 < \alpha = .06$. So, we reject H_0 at 6 percent level
 .

 (b) We have $.02 < p_v = .0186 < .02$. So lowest level of rejection of H_0
 is 2 percent.

2. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \mu = 34 \\ H_A : \mu > 34 \end{cases}$$

So, $\mu_0 = 34$, The sample size $n = 26$
Observed Sample mean $\overline{x} = 37$
Observed sample standard deviation $s = 11$,
$t = \frac{(\overline{x}-\mu_0)\sqrt{n}}{s} = \frac{(37-34)\sqrt{26}}{11} = 1.3906$
This is a Right Tail Test. So,
p-value $p_v = P(t < T) = tcdf(t, 5, n-1)$
$= tcdf(1.3906, 5, 25) = .0883.$

(a) Given Level of Significance is 4 percent. So, $\alpha = .04$.
    We have $p_v = .0883 \not< \alpha = .04$. So, we will NOT reject H_0 at 4
    percent level
    .

(b) We have $.08 < p_v = .0883 < .09$. So lowest level of rejection of H_0
    is 9 percent.

3. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \mu = 20 \\ H_A : \mu > 20 \end{cases}$$

So, $\mu_0 = 20$, The sample size $n = 31$
Observed Sample mean $\bar{x} = 24$
Observed sample standard deviation $s = 8$,
$t = \frac{(\bar{x} - \mu_0)\sqrt{n}}{s} = \frac{(24-20)\sqrt{31}}{8} = 2.7839$
This is a Right Tail Test. So,
p-value $p_v = P(t < T) = tcdf(t, 5, n-1)$
$= tcdf(2.7839, 5, 30) = .0046$.

(a) Given Level of Significance is 1 percent. So, $\alpha = .01$.
    We have $p_v = .0046 < \alpha = .01$. So, we will reject H_0 at 1 percent
    level
    .

(b) We have $.001 < p_v = .0046 < .005$. So lowest level of rejection of
    H_0 is .5 percent.

4. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \mu = 10 H_A : \mu > 10 \end{cases}$$

So, $\mu_0 = 10$. Use TI. The sample size $n = 22$
Observed Sample mean $\bar{x} = 14.4091$
Observed sample standard deviation $s = 7.9502$,
$t = \frac{(\bar{x} - \mu_0)\sqrt{n}}{s} = \frac{(14.4091-10)\sqrt{22}}{7.9502} = 2.6013$
This is a Right Tail Test. So,
p-value $p_v = P(t < T) = tcdf(t, 5, n-1)$
$= tcdf(2.6013, 5, 21) = .0083$.

   (a) Given Level of Significance is 4 percent. So, $\alpha = .04$.
We have $p_v = .0083 < \alpha = .04$. So, we will reject H_0 at 4 percent level

    .

   (b) We have $.005 < p_v = .0083 < .01$. So lowest level of rejection of H_0 is 1 percent.

5. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \mu = 33 \\ H_A : \mu > 33 \end{cases}$$

So, $\mu_0 = 33$. The sample size $n = 19$
Observed Sample mean $\overline{x} = 35$
Observed sample standard deviation $s = 5.6$,
$t = \frac{(\overline{x} - \mu_0)\sqrt{n}}{s} = \frac{(35-33)\sqrt{19}}{5.6} = 1.5567$
This is a Right Tail Test. So,
p-value $p_v = P(t < T) = tcdf(t, 5, n-1)$
$= tcdf(1.5567, 5, 18) = .0684$.

   (a) Given Level of Significance is 4 percent. So, $\alpha = .04$.
We have $p_v = .0684 \not< \alpha = .04$. So, we will NOT reject H_0 at 4 percent level

   (b) We have $.06 < p_v = .0684 < .07$. So lowest level of rejection of H_0 is 7 percent.

6. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \mu = 45 \\ H_A : \mu > 45 \end{cases}$$

So, $\mu_0 = 45$. The sample size $n = 27$
Observed Sample mean $\overline{x} = 42$
Observed sample standard deviation $s = 7.5$,
$t = \frac{(\overline{x} - \mu_0)\sqrt{n}}{s} = \frac{(42-45)\sqrt{27}}{7.5} = -2.0785$
This is a Left Tail Test. So,
p-value $p_v = P(T < t) = tcdf(-5, t, n-1)$
$= tcdf(-5, -2.0785, 26) = .0238$.

(a) Given Level of Significance is 4 percent. So, $\alpha = .04$.
We have $p_v = .0238 < \alpha = .04$. So, we reject H_0 at 4 percent level

(b) We have $.02 < p_v = .0238 < .03$. So lowest level of rejection of H_0 is 3 percent.

9.3   Population Proportion

1. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : p = .5 \\ H_A : p > .5 \end{cases}$$

So, $p_0 = .5$. The sample size $n = 211$
Number of success $X = 119$.
The sample proportion of success $\overline{X} = \frac{X}{n} = .5640$
The Test Statistics $z = \frac{(\overline{x}-p_0)\sqrt{n}}{\sqrt{p_0(1-p_0)}} = \frac{(.5640-.5)\sqrt{211}}{\sqrt{.5*.5}} = 1.8593$
This is a Right Tail Test. So,
p-value $p_v = P(z < Z) = normalcdf(z, 5)$
$= normalcdf(1.8593, 5) = .0315$.

(a) Given Level of Significance is 4 percent. So, $\alpha = .04$.
We have $p_v = .0315 < \alpha = .04$. So, we reject H_0 at 4 percent level

(b) We have $.03 < p_v = .0315 < .04$. So lowest level of rejection of H_0 is 4 percent.

2. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : p = .2 \\ H_A : p > .2 \end{cases}$$

So, $p_0 = .2$. The sample size $n = 276$
Number of success $X = 64$.
The sample proportion of success $\overline{X} = \frac{X}{n} = .2319$

The Test Statistics $z = \frac{(\bar{x}-p_0)\sqrt{n}}{\sqrt{p_0(1-p_0)}} = \frac{(.2319-.2)\sqrt{276}}{\sqrt{.2*.8}} = 1.3249$

This is a Right Tail Test. So,

p-value $p_v = P(z < Z) = normalcdf(z, 5)$

$= normalcdf(1.3249, 5) = .0926$.

(a) Given Level of Significance is 4 percent. So, $\alpha = .04$.
We have $p_v = .0926 \not< \alpha = .04$. So, we will NOT reject H_0 at 4 percent level

(b) We have $.09 < p_v = .0926 < .10$. So lowest level of rejection of H_0 is 10 percent.

3. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : p = .13 \\ H_A : p < .13 \end{cases}$$

So, $p_0 = .13$. The sample size $n = 721$

Number of success $X = 76$.

The sample proportion of success $\overline{X} = \frac{X}{n} = .1054$

The Test Statistics $z = \frac{(\bar{x}-p_0)\sqrt{n}}{\sqrt{p_0(1-p_0)}} = \frac{(.1054-.13)\sqrt{721}}{\sqrt{.13*.87}} = -1.9641$

This is a Left Tail Test. So,

p-value $p_v = P(z < Z) = normalcdf(-5, z)$

$= normalcdf(-5, -1.9641) = .02476$

(a) Given Level of Significance is 4 percent. So, $\alpha = .04$.
We have $p_v = .02476 < \alpha = .04$. So, we reject H_0 at 4 percent level

(b) We have $.02 < p_v = .02476 < .03$. So lowest level of rejection of H_0 is 3 percent.

9.4   Testing Hypotheses on Variance σ^2

1. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \sigma^2 = 196 \\ H_A : \sigma^2 > 196 \end{cases}$$

So, $\sigma_0^2 = 196$. The sample size $n = 27$
Sample variance $s^2 = 324$.
The Test Statistics $Y = \frac{(n-1)s^2}{\sigma_0^2} = \frac{26*324}{196} = 42.9796$
This is a Right Tail Test. So,
p-value $p_v = P(y < Y) = 1 - P(Y < y) = 1 - \chi^2 cdf(0, y, n-1)$
$= 1 - \chi^2 cdf(0, 42.9796, 26) = 1 - .9806 = .0194$

   (a) Given Level of Significance is 5 percent. So, $\alpha = .05$.
       We have $p_v = .0194 < \alpha = .05$. So, we reject H_0 at 5 percent level

   (b) We have $.01 < p_v = .0194 < .02$. So lowest level of rejection of H_0
       is 2 percent.

2. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \sigma^2 = 5 \\ H_A : \sigma^2 > 5 \end{cases}$$

So, $\sigma_0^2 = 5$. The sample size $n = 26$
Sample variance $s^2 = 8.8$.
The Test Statistics $Y = \frac{(n-1)s^2}{\sigma_0^2} = \frac{25*8.8}{5} = 44$
This is a Right Tail Test. So,
p-value $p_v = P(y < Y) = 1 - P(Y < y) = 1 - \chi^2 cdf(0, y, n-1)$
$= 1 - \chi^2 cdf(0, 44, 25) = 1 - .9892 = .0108$

   (a) Given Level of Significance is 4 percent. So, $\alpha = .04$.
       We have $p_v = .0108 < \alpha = .04$. So, we reject H_0 at 4 percent level

   (b) We have $.01 < p_v = .0108 < .02$. So lowest level of rejection of H_0
       is 2 percent.

3. The Null and Alternate Hypotheses are

$$\begin{cases} H_0 : \sigma^2 = 20000 \\ H_A : \sigma^2 > 20000 \end{cases}$$

So, $\sigma_0^2 = 20000$. The sample size $n = 19$
Sample variance $s^2 = 32500$.
The Test Statistics $Y = \frac{(n-1)s^2}{\sigma_0^2} = \frac{18*32500}{20000} = 29.25$
This is a Right Tail Test. So,
p-value $p_v = P(y < Y) = 1 - P(Y < y) = 1 - \chi^2 cdf(0, y, n-1)$
$= 1 - \chi^2 cdf(0, 29.25, 18) = 1 - .9546 = .0454$

(a) Given Level of Significance is 4 percent. So, $\alpha = .04$.
We have $p_v = .0454 \not< \alpha = .04$. So, we will NOT reject H_0 at 4 percent level

(b) We have $.04 < p_v = .0454 < .05$. So lowest level of rejection of H_0 is 5 percent.