Math 790 **Test 5 (Takehome)** Satya Mandal
Fall 05 Each Problem 10 points Due on: October 17, 2005

Unless otherwise stated, $\mathbb{F}$ is a field and $\mathbb{F}[X]$ is the polynomial ring over $\mathbb{F}$.

1. Prove that a polynomial $f(X) \in \mathbb{F}[X]$ of degree n has atmost n roots in $\mathbb{F}$.

2. Suppose I is a non-zero ideal in $\mathbb{F}[X]$. Prove that $I = \mathbb{F}[X]d$ for some $d \in I$.

3. Suppose $f_1, \ldots, f_n \in \mathbb{F}[X]$ and not all of them are zero. Prove the they have a CGD and the two GCDs differ by a unit multiple.

4. Suppose $f_1, \ldots, f_n \in \mathbb{F}[X]$ are all non-zero. Suppose p is a prime element. Prove that if p divides the product $f_1 f_2 \cdots f_n$ then p divides f_i for some $i = 1, \ldots, n$.

5. Prove that any $f \in \mathbb{F}[X]$ has a uniques factorization as
$$f = u p_1 p_2 \cdots p_r$$
where u is an unit and p_i is a prime in $\mathbb{F}[X]$ for $i = 1, \ldots, r$.

6. (About determinants) Let V be a vector space over $\mathbb{F}$ with $\dim(V) = n$. Let $T \in L(V, V)$

be a linear operator. Define determinant of T and prove that it is well defined.