Math 790 **Test 2 (Takehome)** Satya Mandal
Fall 05 Each Problem 10 points Due on: Spet 16, 2005

Unless otherwise stated, F is a field.

1. Let V be a vector space over F and W be a non-empty subset of V. Prove that the following are equivalent:
   (a) W is a subspace of V.
   (b) For $u, v \in W$ and $c, d \in F$ we have $cu + dv \in W$.
   (c) For $u, v \in W$ and $c \in F$ we have $u + v \in W$ and $cu \in W$.
   (d) For $u, v \in W$ and $c \in F$ we have $cu + v \in W$.

2. Let V be a vector space over F and S be a non-emptysubset of V.
   (a) Define the subspace spanned by S. Write $W = Span(S)$.
   (b) Prove that if U is a subspace of V containing S, then W is contained in U.
   (c) Prove

$$W = \{c_1v_1 + c_2v_2 + \cdots + c_nv_n : n \geq 0, c_i \in F, v_i \in S\}.$$

3. Let V be a vector space over F and V is spanned by a finite set $S = \{v_1, \ldots, v_n\}$. Prove that a subset of S will form a basis of V.

4. Let V be a finitel dimensional vector space over F let $S = \{v_1, \ldots, v_n\}$ be a linearly independent subset. Prove that S extends to a basis of V. (*We really do not need to assume that V has finite dimension.*)

5. Let V be a vector space over F and V is spanned by a finite set $S = \{v_1, \ldots, v_n\}$. Prove that any two basis of V have same number of elements. (*We really do not need to assume that S is a finite set.*)

6. Let V be a vector space over F and W_1, W_2 be two subspaces of V. Assume $W_1 + W_2$ has finite dimension. Prove that

   $$\dim(W_1+W_2) = \dim(W_1)+\dim(W_2)-\dim(W_1 \cap W_2).$$

7. Let A, B be two $m \times n$ matrices with entries in F. Prove that A and B have same row space if and only if they are row equivalent.

8. Let $V = F[X]$ be set of all polynomials over F. Prove that, as a vector space, V does not have finite dimension.