Math 790	**Test 4 (Takehome)**	Satya Mandal
Fall 05	Each Problem 10 points	Due on: October 10, 2005

Unless otherwise stated, F is a field and V, W are two vector sapces over F. .

1. Suppose V is a vector space over $\mathbb{F}$ and $W \subseteq V$ is a subspace of V. Prove that annihilator of the the annihilator of W is itself. That is, notationally, prove that $W = W^{00}$.

2. Suppose V is vector space of finite dimension, $\dim V = n$, over $\mathbb{F}$. Let $g, f_1, \ldots, f_r \in V^*$ be linear functionals. Let N be the null space of g and N_i be the null space of f_i.

   Then, $N_1 \cap N_2 \cap \cdots \cap N_r \subseteq N$ if and only if $g = \Sigma_{i=1}^r c_i f_i$ for some $c_i \in \mathbb{F}$.

3. Suppose V is a vector space over $\mathbb{F}$ and $W \subseteq V$ is a subspace of V. Suppose $g_1, \ldots, g_r \in V^*$ forms a basis of the annihilator W^0. Write $N_i = Null(g_i)$. Prove that

$$W = \cap_{i=1}^r N_i.$$

4. Suppose V, W be two finite dimensional vector spaces over $\mathbb{F}$. Let $T : V \to W$ be a linear transformation and $T^t : W^* \to V^*$ be the transpose. Prove that $rank(T) = rank(T^*)$.

Also prove that for a $m \times n$ matrix A with entries in A, we have $row-rank(A) = column-rank(A)$.

5. Suppose V is vector space of finite dimension, $\dim V = n$, over $\mathbb{F}$. Define the map

$$\varphi : L(V, V) \to L(V^*, V^*)$$

by $\varphi(T) = T^t$. Prove that φ is an isomorphism.