Theorem 0.1 *Let V be a vector space over a field F and $V = Span(e_1, e_2, \ldots, e_n)$. Suppose $m > n$ is an integer and $v_1, \ldots, v_m$ be m elements in V. Then $v_1, \ldots, v_m$ are linearly dependent.*

Proof. We have

$$\begin{pmatrix} v_1 \\ v_2 \\ \cdots \\ v_m \end{pmatrix} = A \begin{pmatrix} e_1 \\ e_2 \\ \cdots \\ e_n \end{pmatrix}$$

Where $A = (a_{ij})$ is an $m \times n$ matrix with $a_{ij} \in F$. Now the homogeneous system of equations

$$(X_1, \ldots, X_m)A = (0, 0, \ldots, 0)$$

has n equations in m unknowns. Since $m > n$, the system has a non-zero solution. That means there is a row vector $(c_1, c_2, \ldots, c_m)$, not all $c_i = 0$ and

$$(c_1, c_2, \ldots, c_m)A = (0, 0, \ldots, 0).$$

Therefore

$$(c_1, c_2, \ldots, c_m) \begin{pmatrix} v_1 \\ v_2 \\ \cdots \\ v_m \end{pmatrix} = (c_1, c_2, \ldots, c_m)A \begin{pmatrix} e_1 \\ e_2 \\ \cdots \\ e_n \end{pmatrix} = 0$$

Hence $\sum_{i=1}^{m} c_i v_i = 0$ and $v_1, v_2, \ldots, v_m$ are linearly dependent.