Math 996: A course on Quadratic Forms

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The following is an overview of the course on Quadratic Forms.

In this course, we study quadratic forms.

1. Given a field F, a quadratic form q is a homogeneous polynomial of degree two. So, it looks like

$$q = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} X_i X_j \qquad\qquad a_{ij} \in F.$$

   We will always assume $1/2 \in F$.

2. By a change of variables, any quadratic form is diagonalizable. That means, after change of variables,

$$q = a_1 X_1^2 + a_2 X_2^2 + \cdots + a_n X_n^2 \qquad\qquad a_i \in F.$$

3. In my view, after linear polynomials, simplest mathematical objects are the quadratic forms, which we will study.

4. **Overview:** One of the simplest (or trivial) quadratic form is $f(X, Y) = X^2 - Y^2$, which is called a hyperbolic form. Given a field F, we will define Witt group $W(F)$ of F. Generators of $W(F)$ are the quadratic forms (up to isometry), and hyperbolics are treated as zero (or trivial). These groups $W(F)$ will be among the main objects of our study.

5. **Background Needed:** Some familiarity with fields. Among the fields, we will consider are $\mathbb{Q}, \mathbb{R}, \mathbb{Z}_p$ where p is a prime number.

6. **Textbook:** "*Introduction to Quadratic forms over Fields*" by T. Y. Lam.

   I will probably be able to finish upto Chapter VI, which deals with computing the Witt group $W(\mathbb{Q})$ of $\mathbb{Q}$.

7. **Lecture Notes:** As always, I will be to able provide complete online lecture notes for the whole course (in pdf).